DOCUMENT RESUME

ED 200 394 SE 033 235

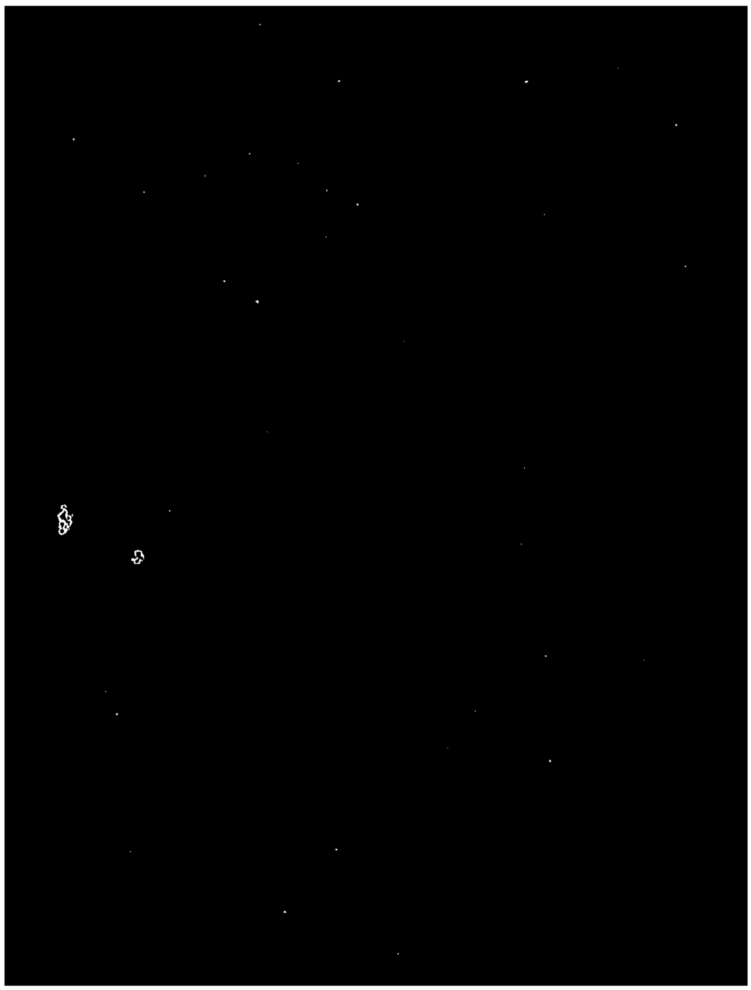
AUTHOR Barclay, William L., III; And Others
TITLE Teacher's Guide for Senior Balancing.
INSTITUTION Elementary Science Study, Newton, Mass.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 70
NOTE 92p.; Photographs may not reproduce well.

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Elementary Education; *Elementary School Science; Instructional Materials; Interdisciplinary Approach; Mathematical Applications; *Physical Sciences; *Science Activities; *Science Course Improvement Projects; Science Curriculum; Science Education; Science Instruction; *Teaching Guides
IDENTIFIERS *Elementary Science Study

ABSTRACT

This guide provides opportunities for students to investigate several aspects of balancing including weighing, equilibrium, symmetry, gravity, and center of gravity, bringing mathematics and science together in activities involving graphing, inequalities, and number relations. This is divided into five major sections: (1) introduction, (2) beginning activities, (3) background for continuing investigations, (4) extensions, and (5) an appendix. The introduction provides information regarding teaching suggestions, scheduling, ages, and materials supplied. Beginning activities include early work, problems, troubleshooting, descriptions, numbering boards, and problem cards. The background for continuing investigations includes activities for predicting, weighing, off-center boards, and problems without solutions. The extensions section includes activities involving what makes the balance work and challenging problems. The appendix includes information on making balancing equipment and an example of unexpected happenings in the classroom. (DS)

teacher's guide for SENIOR BALANCING



The Senior Balancing Unit

Teacher's Guide for Senior Balancing
Problem Cards for Senior Balancing
Teacher's Kit for Senior Balancing
6-Student Kit for Senior Balancing

Related Units

Primary Balancing
Mobiles

17588
1 2 3 4 5 6 7 8 9 10 EBEB 79 78 77 76 75 74 73 72 71 70

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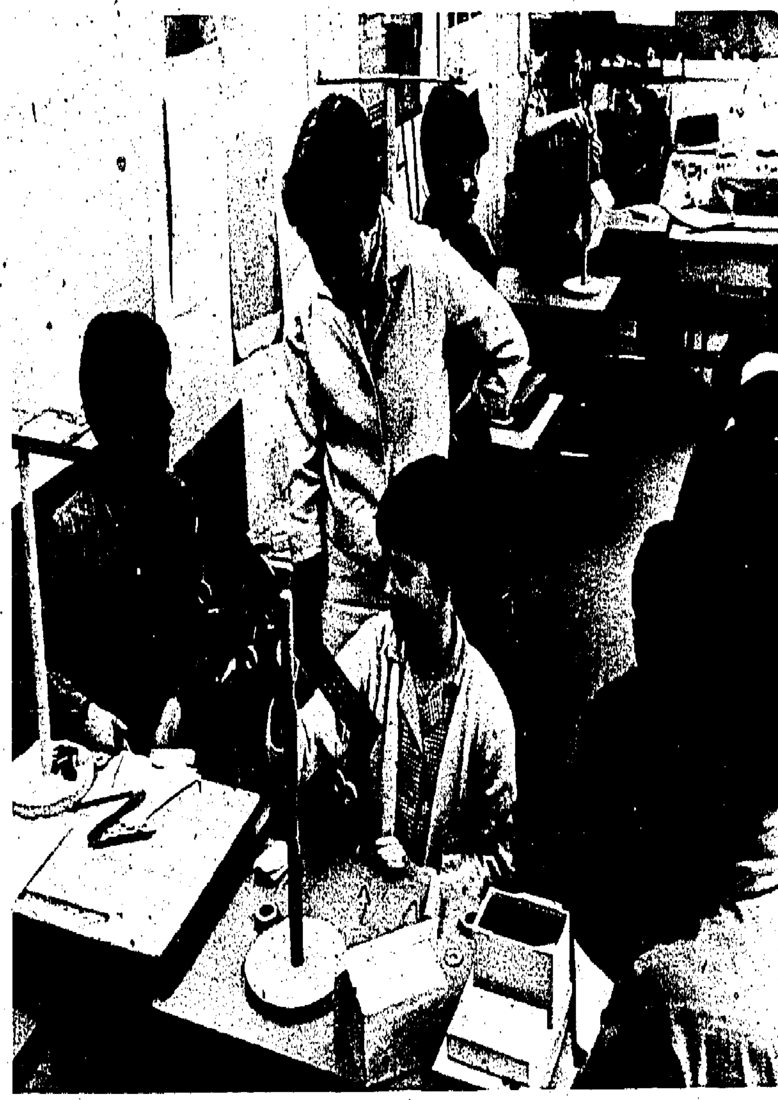
Some Suggestions for Teaching

To gain insight into the subject of balancing, children need time to explore many arrangements with the basic equipment. They should develop a "feel" for when things will and won't balance and an idea of the kind of work they can do with the basic materials.* You serve as a source of guidance and support when children begin to draw conclusions and make generalizations and predictions from their own observations.

*Children who have been exposed to the activities in the Elementary Science Study unit THE BALANCE BOOK have been given a chance to work with large balancing equipment, with pan balances, and with some of the same type of equipment used here, and have developed a sense of when things will balance. THE BALANCE BOOK and Materials for PRIMARY BALANCING are available from the Webster Division of McGraw-Hill Book Company, Manchester Road, Manchester, Missouri 63011.

There is no one preferred way to teach this unit. There have been successful classrooms in which the teacher has merely set aside a table of balancing equipment. She will, perhaps, ask an occasional question, add a new bit of equipment, or suggest a new problem or a new approach to an old problem by giving the children one of the *Problem Cards** supplied or one she has designed.



**Problem Cards for Senior Balancing* to go with this unit are available from the Webster Division of McGraw-Hill Book Company. See page 21 for a description.

SENIOR BALANCING has also been done by an entire class, at specified times and with specific activities and problems planned for each session. Satisfactory results can be achieved this way, too. Between these extremes lies a range of other possible ways to present this material.

One child's work may lead the activities of the class in a new direction, possibly only vaguely related to the subject at hand. These classes will often prove to be among the most exciting and valuable to you as well as to the children. (For an account of a class that discovered an interesting sidetrack, see Appendix A.)

Naturally, you will select the style that suits your own objectives and personality, and you will probably alter your approach a little bit each time you use this material.

On the first trial, most teachers are working through this material along with the children and need some guidelines to follow. "Beginning Activities" attempts to provide such a path through the material. In subsequent years, you should not need to follow the suggestions so closely.

The teaching pattern described here suggests a sequence for distributing the various items of equipment and also provides you with some notes on the types of activities that can be expected to occur at each stage of the game.

Under "Background for Continuing Investigations," you will find discussions of specific topics that you will want to explore on your own. You can work through these sections in any order, selecting those that are pertinent.

Scheduling

If the class works as a whole, you can spend as few as ten class periods or as many as thirty on balancing. Teachers have found it best to allot two or three periods a week, each period of at least forty-five minutes, to this work. On some days the children's interest may keep them working longer at balancing; on other days a shorter period may be advisable.

Of course, if the children work at SENIOR BALANCING individually or in a small group, you can set aside time and space for this as you do for other activities. Approached in this way, as an elective activity, the unit can continue throughout the school year.

You may find, too, that the balancing equipment will be called for time and again as the children feel the need for it in connection with their other studies.

Ages

This unit has been used successfully with children in grades three through eight; the majority of trial classes have been in grades five and six. Children of any age tend to do the same sort of simple work with the balance equipment in the beginning. For children who have already done balancing work in the primary grades, this unit provides a continuation, enabling them to explore balancing in more analytic and mathematical ways.

Using the materials However you choose to teach SENIOR BALANCING, you will need one *Teacher's Kit*. In addition, one *6-Student Kit* will suffice if children are to work on balancing individually at times of their own choosing. If the whole class will be working on balancing at one time, order enough *6-Student Kits* to supply each child with his own basic equipment. You will also need one set of *Problem Cards for Senior Balancing*.

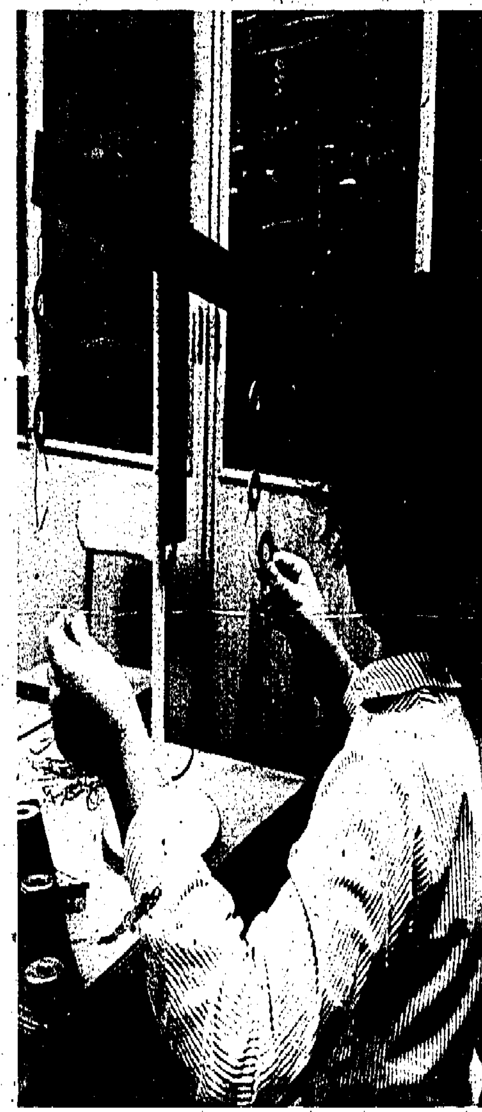
When the whole class is working together, it is helpful to have available individual containers, such as plastic freezer containers or heavy plastic bags, in which each child can store his washers and paper clips. Some teachers have found it helpful to deaden the noise of dropped washers by providing children with newspaper or place mats to work on. Working directly on the floor also reduces clatter.

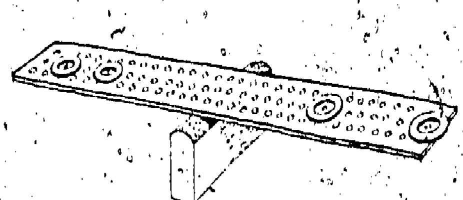
Many teachers have found it helpful to provide a package or two of wire shower curtain rings. The children use these rings when they wish to hang large numbers of washers on the board without dropping them.

Children like to fashion their own balancing equipment also, using materials from regular school supplies or items brought from home. (See Appendix B for suggestions for balances children can easily build.)

Two ways to set up the board For later work, each child gets a wooden fulcrum with a curved top that is coated with sand. This makes it possible to use the pegboard both as a flat balance beam, with the washers on top of it, and as a hanging beam, from which the washers are suspended by paper clips.

Some children develop a strong preference for one form of balancing over the other, but it is worth pointing out that most problems can be done both ways, and it is useful to suggest to a child who is having trouble with a problem that he shift from the hanging board to the flat board.



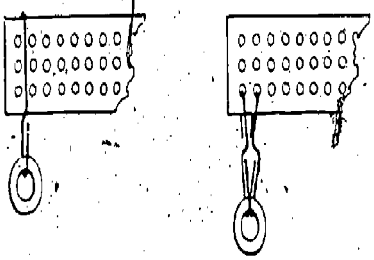


For most work, the board is hung on the stand and the washers are suspended from it by paper-clip hooks. In this case, there is no question about which hole the washer weight is placed at, and the washers stay where they're put. Even when the board is very heavy with washers, they won't fall off, and, with the board hung on the stand, it is easy to see how much the board is tilted.

18

19

Some of the diagrams show a board on the block fulcrum. There are advantages to using the board flat in some situations, in addition to the contrast it provides with the board hung on the nail fulcrum. For example, you can easily place a washer halfway between two holes. To do this on the hanging board entails putting the paper clips over the top of the board or hanging a washer on two clips suspended from adjacent holes.



The flat board is also useful when you want to slide the weights gradually along the board until you find their proper position, or when you want to shift washers by a series of symmetrical moves as on pages 23 through 25.

It can also be used with small blocks and other kinds of weights that can't be hung from clips.

The flat board does, however, have several disadvantages. (For this reason it is suggested that the sandpaper block be left out of the pupils' boxes for the first few sessions.) For one thing, the board is easily tilted so that one end touches the table top, which makes it difficult to see how much adjustment is needed to achieve balance. For another, the washers tend to shift (especially if one end of the board hits the table top), further complicating attempts to balance the board.

Using the double-size board The double-size board is useful when children wish to demonstrate a problem for the rest of the class. Some teachers have put a cup hook or nail into the wall near the chalkboard, so that the board is easily seen during a discussion. Others hang the double-size board from a student balance stand placed on a high table or a pile of books so that everyone can see what is going on. The 1/4" tape may be used along the bottom edge to make the board stand out against a dark background and can also be used for numbering.

Occasionally, teachers have used this board to pose a problem, but it should not be considered primarily a teacher's tool. Experience has shown that children come to understand concepts better when they themselves work through the ideas.

Trying problems with this board gives children experience using a board of a different length. They may not see, at first, that the length of the board is unimportant, and they may need this opportunity to gain extra perspective. The half-length and quarter-length strips in the *Teacher's Kit* can be used as short balance boards.

Since the tilt of the double-size board is easy to record on the chalkboard, this equipment may be useful when the children are investigating sensitivity and the effect of one paper clip. The regular length boards can also be used for comparison.

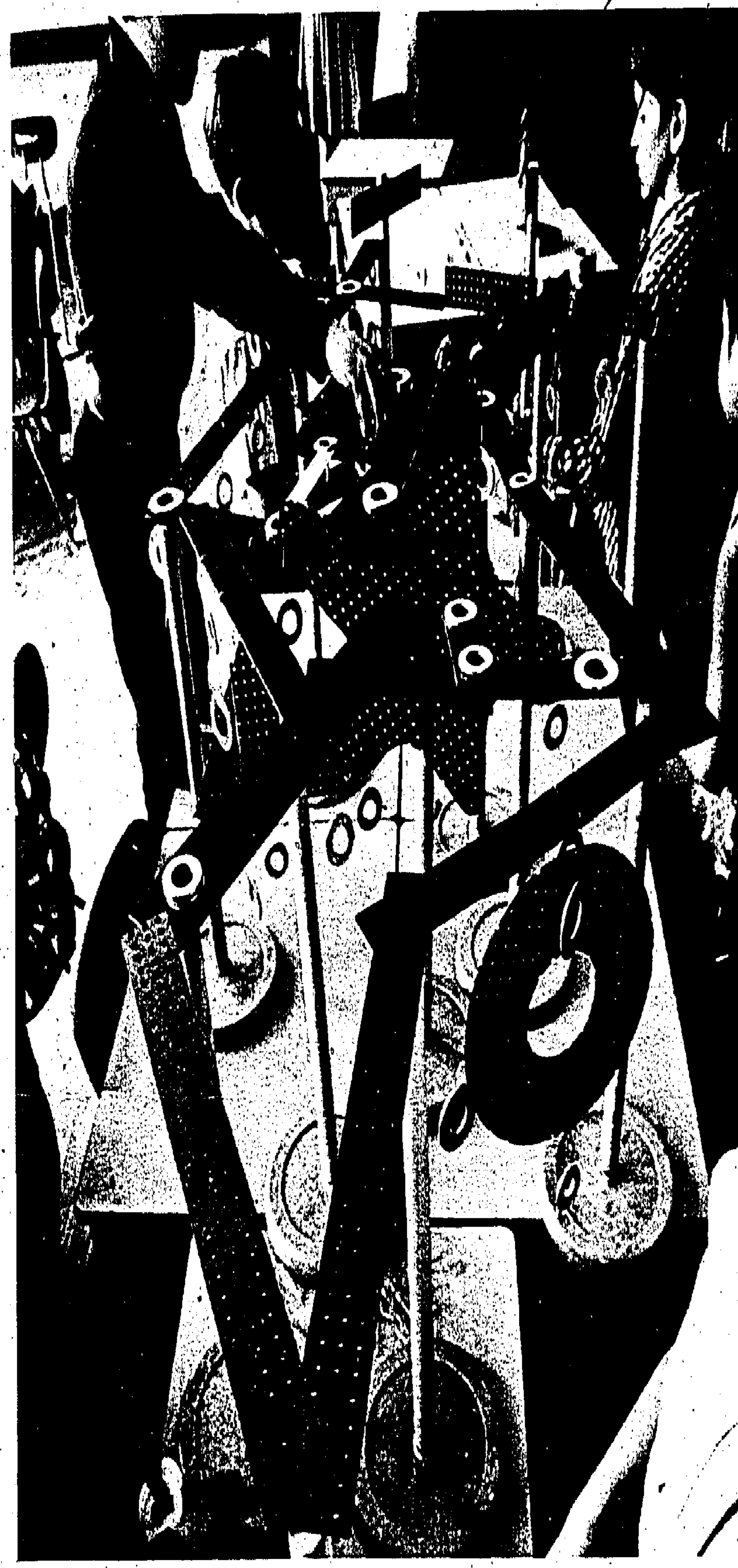
Other materials Items which you can obtain easily are not included among the materials supplied. Modeling clay is used in many activities, because it can be easily shaped, cut, molded, and inscribed with pencil markings and numbers. Where it is not available, teachers have substituted materials such as gum erasers or lengths of string or wire.

In some classes, small blocks have been used on the flat beam, and the children may suggest trying sugar cubes, dominoes, *Attribute Blocks**, or Cuisenaire rods.

Some cards will require special materials, which you or the children can provide.

*Available from the Webster Division of McGraw-Hill Book Company.

These photographs were taken during the early stages of SENIOR BALANCING classes. They give a flavor of the varied activities children sample with the balancing equipment. Later, children may pursue some of these activities in greater depth.



As you look arour
children involved in
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Symmetrical arrangen
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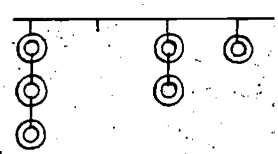
something like this:



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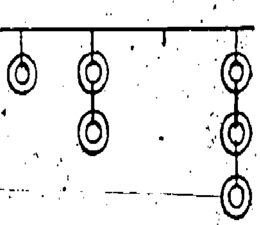
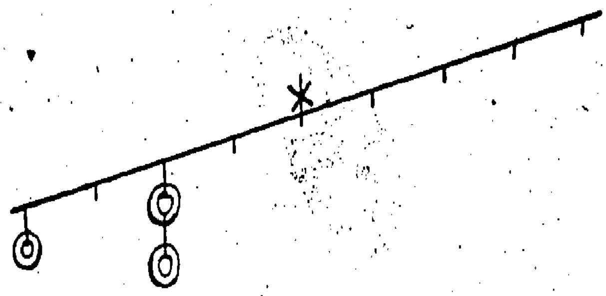
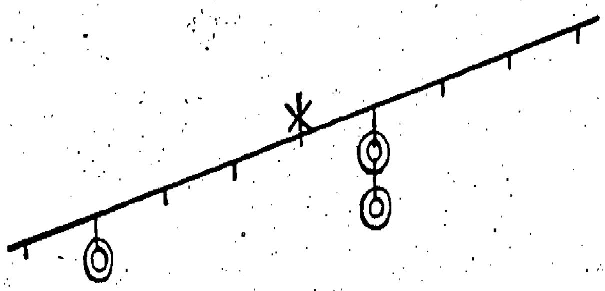


This sort of arrangement can be turned around into a problem: Can you add washers to this board to make one side the mirror image of the other? Don't move the washers already there.



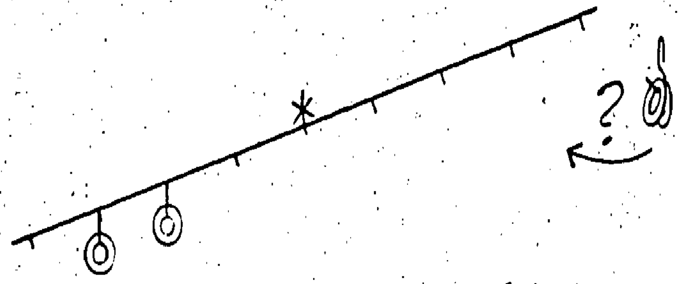
How about this arrangement?



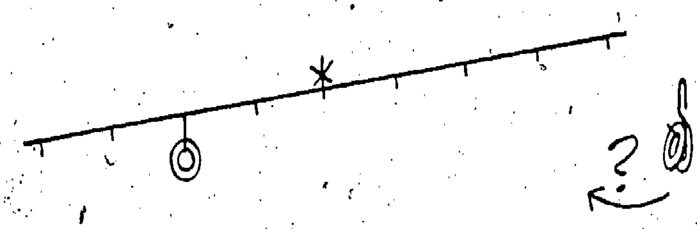
Moving away from symmetrical arrangements to other ways of balancing the board.

Problem:

Can you make the board balance by adding two washers on the same paper clip?



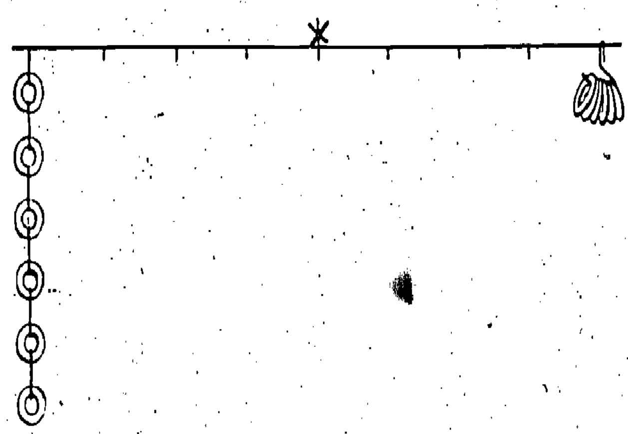
This perhaps leads on to *balancing one washer with two.*



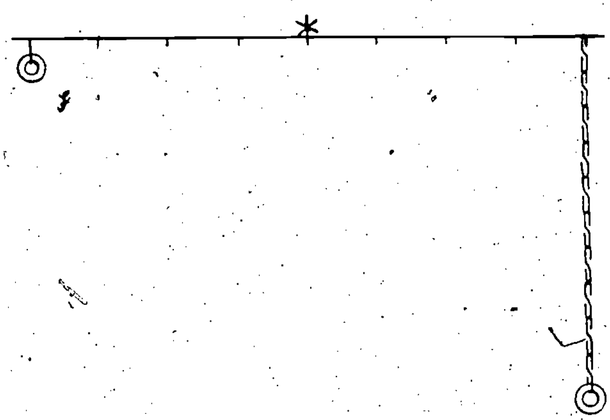
(These problems are discussed in "Predicting," p. 23.) Many children balance washers with long *chains of paper clips.*

Problem:

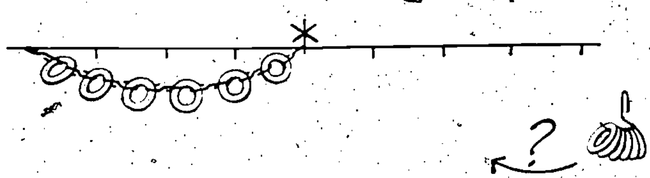
Will a chain of six washers balance six washers hung on one paper clip? (What about the extra paper clips?)

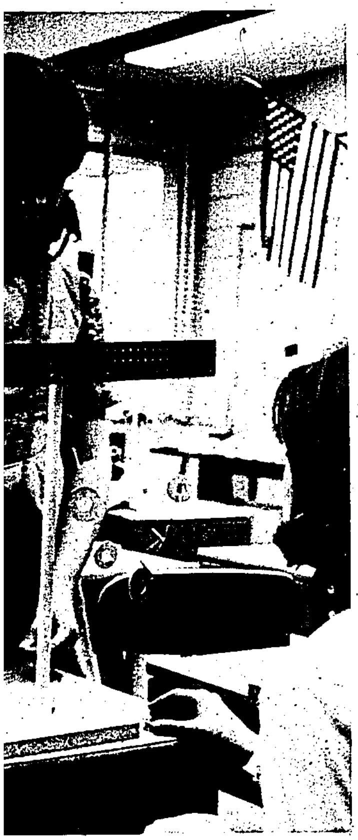


Problem:
Will a washer hung at the bottom of a paper-clip chain have the same effect as one hung at the top?

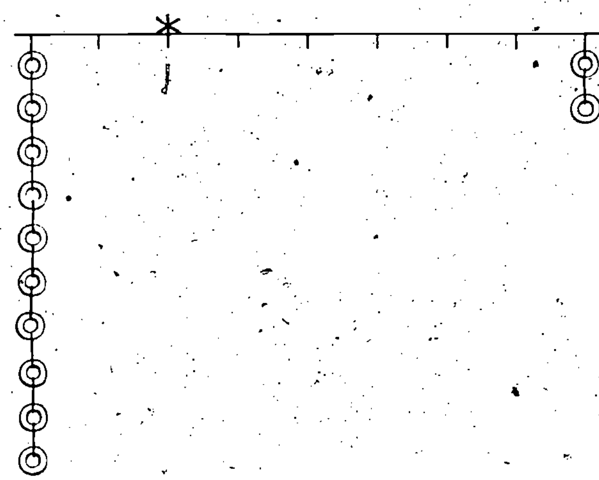


Problem:
Where can you put six washers on one paper clip to balance a chain of six washers?

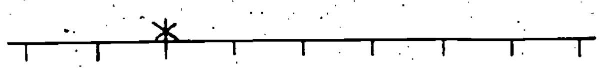




Balancing the board when the nail is off-center— a few washers out on the long side will balance a great many on the short side, an intriguing situation.

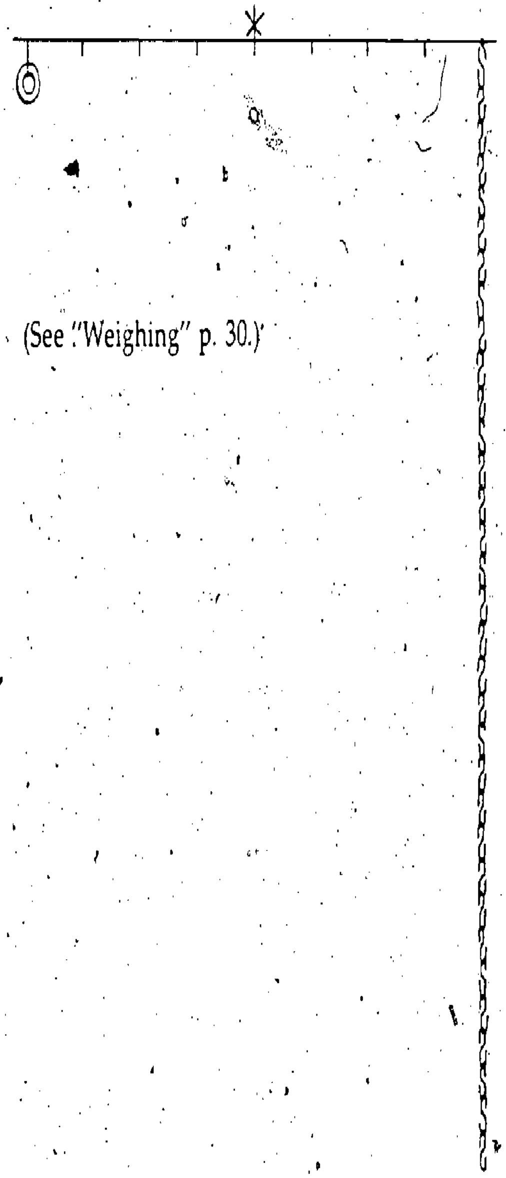


Problem:
If the board is supported near one end, how many washers will be needed to make it level?



(See "Off-center Boards," p. 33.)

Using the board to weigh the washers—one girl did this: Using the pegboard as an equal-arm balance, she hung a single washer on a paper clip from the end hole on one side and kept adding paper clips to the end hole on the other side until the board was balanced. It took 33 paper clips. She announced that the washer weighed the same as 32 paper clips.



(See "Weighing" p. 30.)

When can the board "feel" small differences in weight? One student announced at the end of the first class that when there were no washers on the board, one paper clip made it tilt a lot; after he had hung many washers on it, one paper clip didn't have any effect. He found this out in the course of balancing the board by itself while he was trying out the equipment for the first time. Children often want to investigate this sort of thing more carefully later on.

Problem:
What's the lightest thing you can weigh with the board?

Problem:
What difference does it make when you put the nail in the middle-row center hole instead of the top-row center hole?

(See page 37 and following pages.)

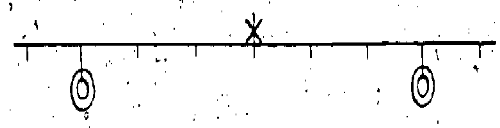
Some Useful Problems

The previous section indicated the wide variety of problems that children elect to tackle at the very beginning of their work with the balances.

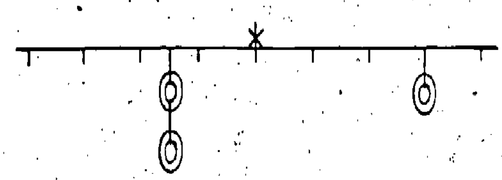
Here are some additional door-opener problems.

1. Hang one washer somewhere on the board. How many ways can you find to balance it? Here are some of the possibilities:

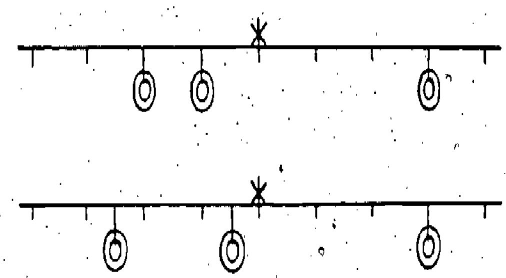
With one washer:



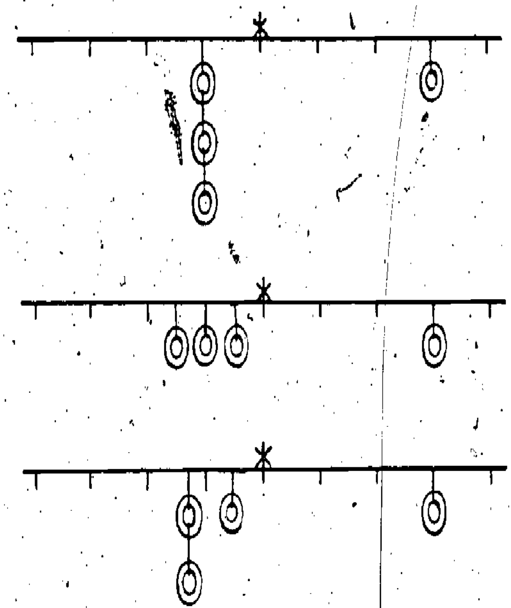
With two on one paper clip:



With two on separate paper clips:



With three washers:

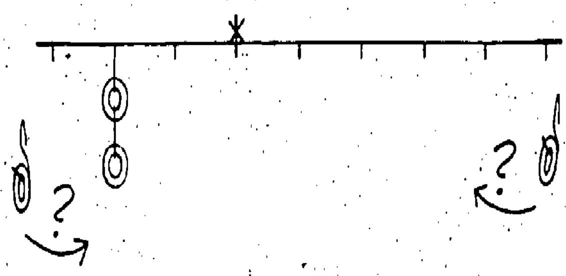


Many children think about these setups in terms of fractions: "The single washer is a third as many as the group of three, and the three go a third as far out."

2. Start with two washers balancing two washers (this problem may be done more easily with the board flat on the fulcrum).

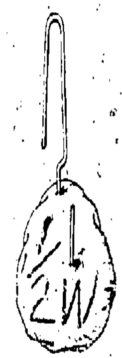
Spread out one pair, leaving the other pair where it was. Find several positions for the spread-out pair, and then reverse the procedure, piling the pair up again.

3. Add washers to an already balanced off-center board.



Where can you add two washers to keep it balanced? Children who have not yet tried working with off-center boards may find that their first guess—"at the ends"—leaves them puzzled, and this starts them thinking about off-center balancing in a way that will get them going on a worthwhile path.

4. Can you make a lump of modeling clay that will weigh half a washer? Some children enjoy working with clay, and they may go on to make a whole series of lumps that are fractions of a washer in weight. The clay can be molded around a paper clip, and its value (such as "1/2 washer") scratched on it with a pencil point.



Does changing the shape of a clay weight affect the balance of the board?

These problems are merely a selection. You may devise others as they are needed. It is not intended that every child do every one. Some will have gone beyond them, and some will not be ready for them. A key problem can be a question which has stumped a child or a puzzling part of balancing that you feel could be worked out by a single child or by the group.

Troubleshooting—When the Board or the Student Doesn't Work

There are times when the board seems to balance and you think it shouldn't, or when it doesn't balance and you think it should. Encouraging the student to go ahead and find the trouble may be worthwhile for him, yet it helps to know what's wrong yourself. Here is a list of typical mechanical difficulties:

Friction—Are the washers hitting or scraping against something? Friction can keep the board from swinging freely.

Fulcrum or Washer Position—Are the fulcrum and washers positioned where the child thinks they are? (He may have counted wrong.) If not, all predictions will go awry.

Unequal Weights—Are washers of the same apparent size the same weight? Try interchanging two washers from opposite sides of the fulcrum. If this changes the balance, one of the washers may be off-weight.

An Uneven Board—Does the board balance at its mid-point without any weights attached? Sometimes one end of a board is heavier than the other.

Also see the section, "What's Balancing What?" on page 27 for a discussion of ways to analyze arrangements that are not balanced.

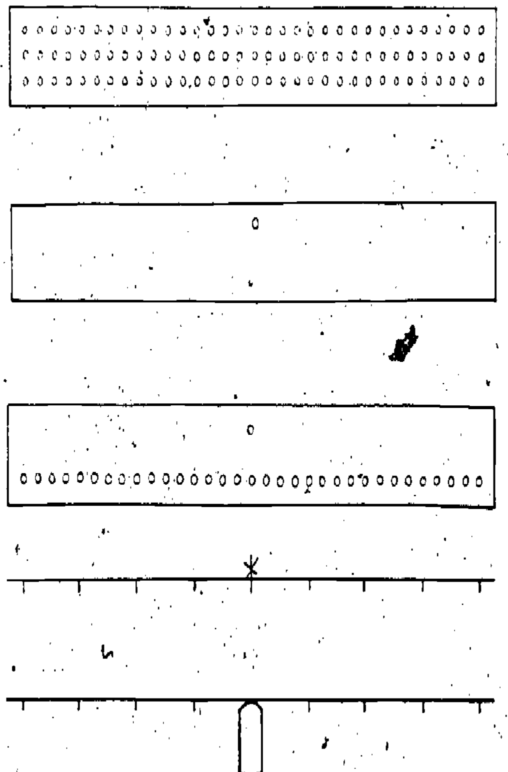
Far more frustrating than an equipment difficulty can be a bored and non-participating student. It does no good to force a student to work with balancing if he is dead set against it, but students often become tempted by one or another of the activities and thus begin to explore further work with the balance.

The cards may act as a stimulant to get a student working. For some children, partnership with a student who is already intrigued works.

Descriptions

During their early explorations with a balance, students often try a great variety of activities. Once they have gained a basic familiarity with a balance, they can go back to a situation, such as balancing an off-center board or weighing light objects, and look at it more carefully. In the process, they must refine their tools of analysis. The following sections discuss numbering the board and ways of describing a balance situation in drawings, words, or numbers. With these skills, children can look at situations such as weighing, predicting, and off-center boards with greater care. These closer looks can lead to a better understanding of balance and balances.

Drawings There are many ways to draw a balance board. Here are five:

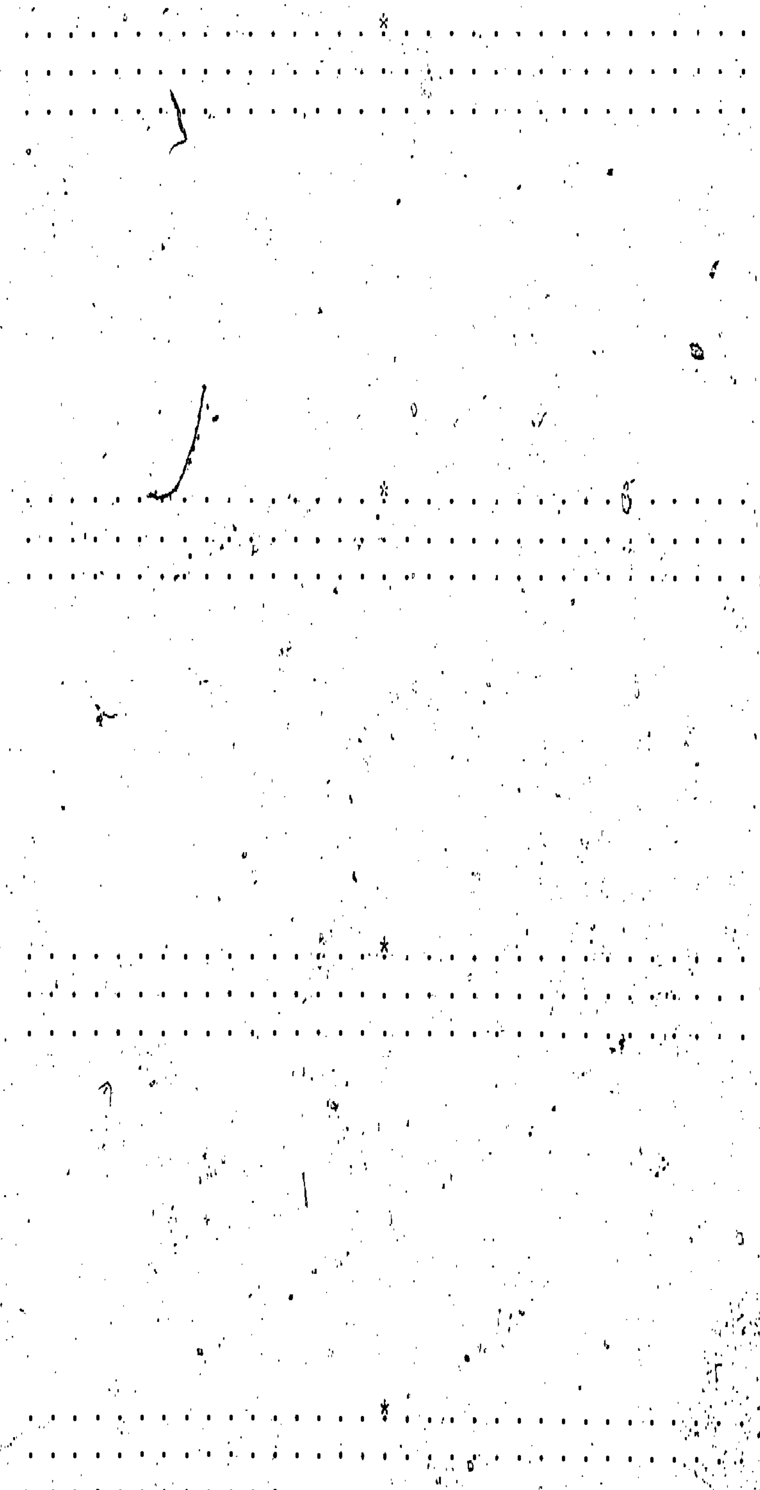


The last shows the board flat on the sandpaper fulcrum.

The last two drawings are the type used throughout this *Guide*.

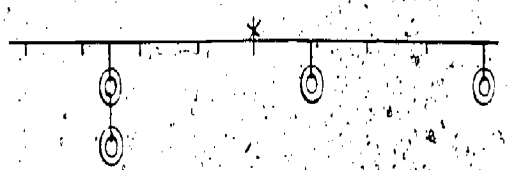
Children use many styles in their drawings, and you should encourage originality rather than conformity. Some children will need a bit more help, however. Dot diagrams, such as those following, can easily be typed onto a ditto master and then duplicated for class use. Although this does not encourage originality, it does help children to record balancing situations quickly and accurately.

Worksheet for Balancing



(* indicates center hole)

Word and number descriptions Another way of recording balancing discoveries is by means of word and number descriptions. Early in the game, a child might describe this board,



somewhat like this: "Two washers, 10 spaces from the nail, balance one washer, 4 spaces, and one washer, 16 spaces from the nail."

The need to simplify this notation may lead to something on this order:

2 @ 10 balances 1 @ 4 and 1 @ 16

which can be further refined to

2 @ 10 = 1 @ 4 + 1 @ 16

or

$2 \times 10 = 1 \times 4 + 1 \times 16$

The final version is not only a shorter form, but also a more sophisticated way of describing the balancing arrangement. If the child has come this far, he has begun to have an idea of a numerical connection between distances and numbers of washers.

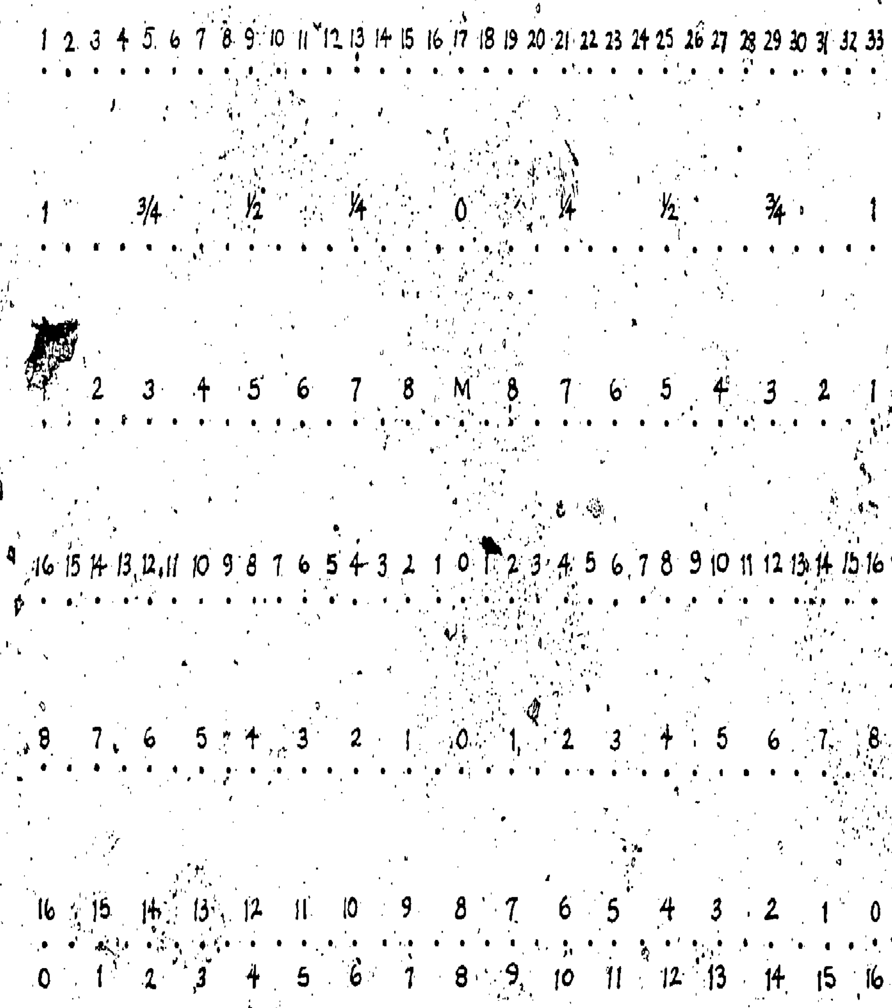
It is important, however, that each child be allowed to arrive at his own form of notation.

The numbers should be used for expressing well-understood physical situations. Insistance on abstract notation can lead to meaningless exercises in arithmetic.

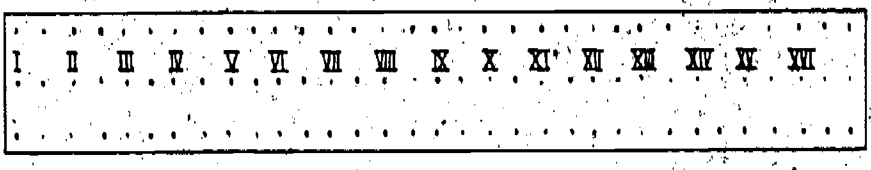
Numbering the Board

After about six or seven sessions, when the class is beginning to get a feeling for the numerical relations involved in balancing, and the children are getting tired of counting holes, the 1/4" tape can be made available to facilitate putting identifying marks or numbers on the board.

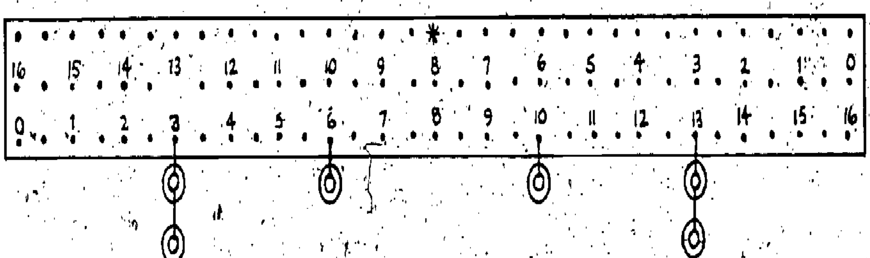
Children typically devise many ways of numbering the board, such as:



One girl even numbered hers using Roman numerals:

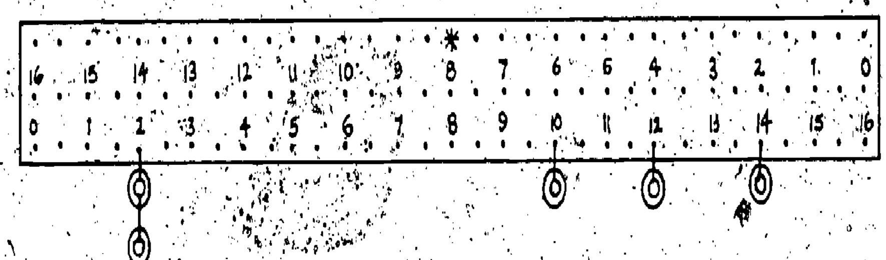


As a child better understands the balancing relations of distances and numbers of washers, a fulcrum-centered system of numbering may appeal to him more. Still, children who count in from the ends can often show that their system allows them to predict. For instance, for symmetrical arrangements–



2 in 3 plus 1 in 6 balances 2 in 3 plus 1 in 6
$(2 \times 3) + (1 \times 6) = (2 \times 3) + (1 \times 6)$.

For non-symmetrical cases, the system is not as helpful:



The numbers the child has put on his board may serve him well for describing how many washers he has, and at which positions, but if he wishes to make predictions or mathematical statements, he must devise some system that will allow him to describe the two sides of the board in terms that simplify the mathematics.

It is essential that the children devise these methods themselves, and their own choice of a method and justification for their preference are more important than the selection of a single "best" way to do it. Later on, they will find it helpful to agree on a common method of numbering for use in class disscussions.

Keep the 1/4" tape available, in case a child wishes to change to a different system for his board. His first method may have proved unsatisfactory to him, or he may want to number the board for a special purpose, such as some off-center experiments.

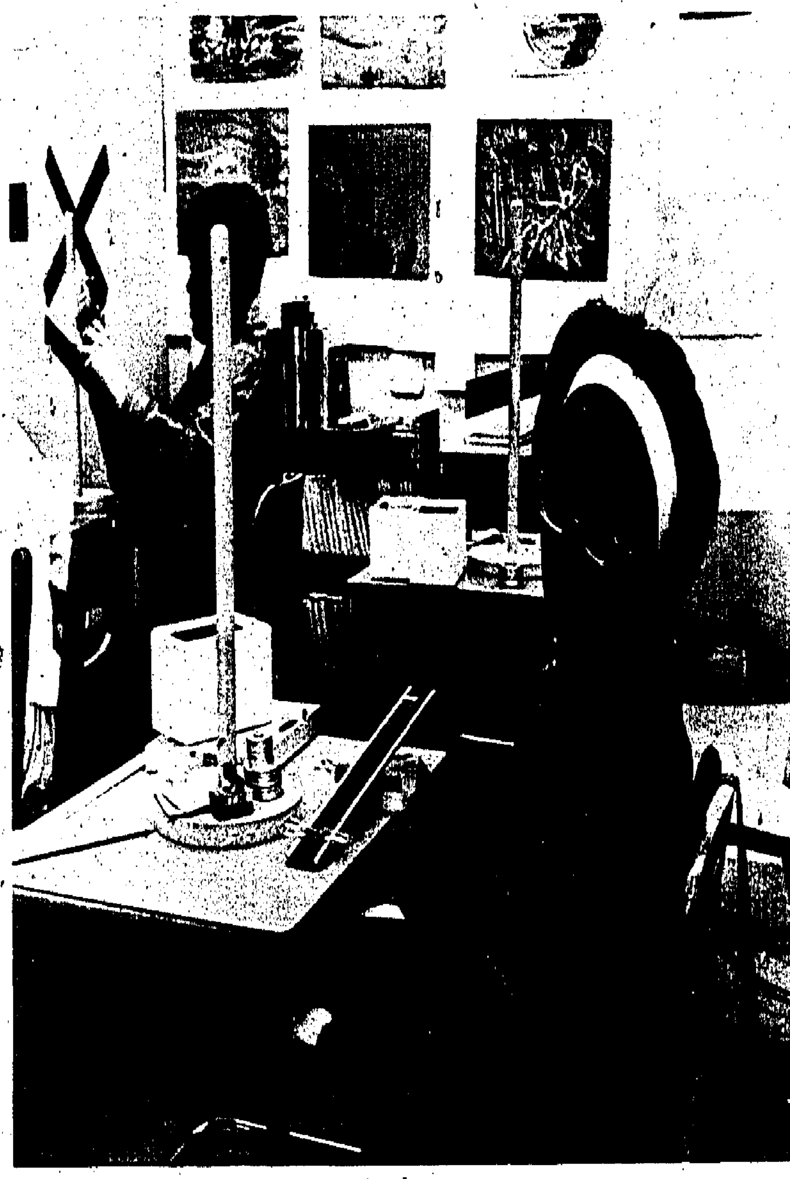
Problem Cards

After about five or six sessions, you may decide that the children are ready for something new and introduce the *Problem Cards*. Most teachers simply choose a location for the cards, and—by way of introduction—tell the class that the cards offer new things to think about. The sixty-three cards are grouped under headings that correspond to sections of this *Guide*. In some cases, there are sequences of cards that give a progression of problems on a particular topic. They are numbered for reference only, and their numbers appear at the beginning of the appropriate section, but, in general, the cards need not be arranged in any special order.

The choice of whether or not to do any particular card or cards (or any cards at all) is usually left up to the child, but those who choose to try the cards should be discouraged from competing to complete "the most" or "the hardest" cards.

You will find it helpful if you take a little time to become familiar with the types of cards in each of the different categories. Then you can easily choose one to help a child pursue the path he has begun or to suggest a new activity to a child who wants something different to try.

The first three cards were written by children in trial classes. Your students may want to try them. You might keep a supply of blank cards available for those who wish to write their own cards. Children usually enjoy trying problems made up by their friends.



Background for Continuing Investigations

The beginning activities already described open the way for a great variety of further activities. Many such activities are discussed here. They are not intended to be done in sequence, nor should any one student or class necessarily do them all.

When you have time, or when a particular related problem has come up, set up your board and work through one of these chapters. Going through a problem on your own will enable you to guide a child who is trying to work through his own balancing problem.

Moving, Piling Up, and Spreading Out

Problem Cards R1–R6

This is a collection of simple problems, all easily accessible to children and all involving central ideas of balancing. If you are not a balance-board expert, you should read this section with a balance and washers at your side. Only by setting up and solving these problems yourself are you likely to understand them. If you spend some time working with a balance board, you will be able to deal effectively with children's questions. Now the problems:

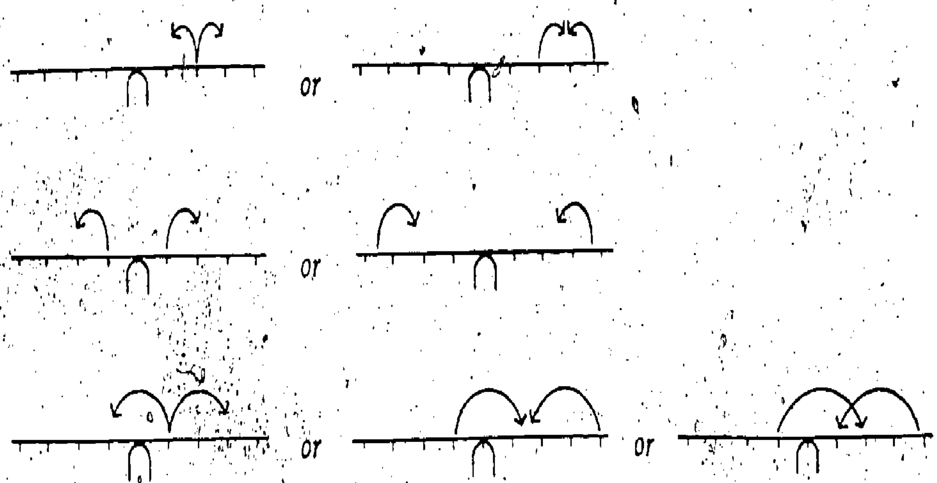
1. Balance the board, using five single washers. Move one washer six spaces toward the fulcrum. Can you rebalance the board by moving only one washer?

2. Start again, this time with the board balanced, using two washers in a pile on one side and three in a pile on the other side. If you move the three-pile four spaces, what do you have to do to rebalance the board? Try other examples, starting with piles of washers.

3. Go back to a pile of two washers balancing a pile of three washers. Investigate ways of spreading out the two-pile washers that do not disturb the balance. What about the three-pile washers? (These can be spread out by leaving one washer in the original position or by moving all three to new positions.)

4. The opposite of spreading out piles of washers is piling washers up. Balance the board with an even number of washers, no two at the same place. By moving two washers at a time, it is possible to rearrange all the washers into two piles that will balance. Be sure you can do this for any initial arrangement of washers. Is it possible, starting with an odd number of washers? (Yes.) Can you get all the washers into a single pile and have the board balance?

In all these problems, there is a basic symmetry to the balance-preserving motions, and you will find this a help in thinking about possible solutions. One way of looking at the problem is to say that every washer moved to the right must be balanced by an equal move of another washer to the left. When unequal piles are involved, the larger pile doesn't need to be shifted as much, but you still move one pile to the right, the other to the left. There is even a right "feel" to the moves that preserve balance – try moving the washers with your eyes closed.

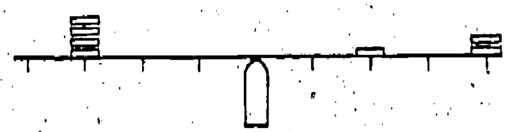


There is also an analytic aspect to these moves – to any balance situation – and this is discussed in the next section.

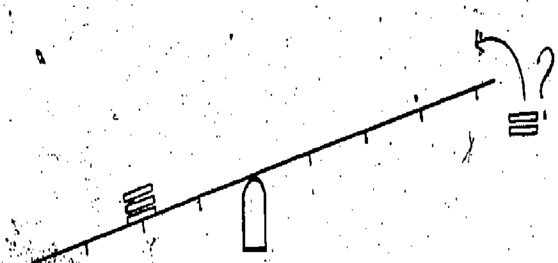
Predicting *Problem Cards O1–O10*

The idea of predicting is important in science, and being asked to predict is fun for children. Initially, a prediction is often just a guess. As you gather experience through trial and error, you become more and more accurate. Early success in predicting often results from a "feel" for balancing. Only much later do you become analytical and verbal.

With balancing, there are innumerable opportunities for predicting. Given an arrangement of washers on a board, will it balance? Does this, for example, balance?



Given a bunch of washers on an unbalanced board, where should you place more washers to make it balance? For instance:



Typically, your ability to predict will be initially manipulative, later conceptual—at first in your fingertips (in knowing where to put two washers to balance one, for instance) and only later transferred to your head as a conscious rule. In this situation, understanding generally builds from simple cases to more complex cases, from balancing symmetrical arrangements to asymmetrical cases, and on toward a broad generalization. You, as a teacher, can ask questions and pose problems to help a child find relationships that underlie balancing experiences, but always your emphasis should be to steer the child into working with the equipment to find his answers.

Here are some predicting games children can play. In one, a child places washers on the board and then specifies the number of washers someone else must use to balance the board. The situation can be as simple or as complicated as the child wants to make it. Variations of arrangements of the washers and of the rules of the game should be encouraged.

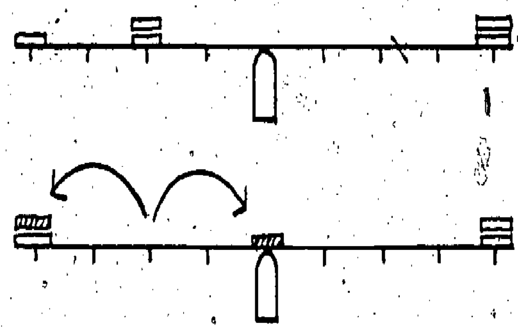
Another game involves predicting whether or not a given arrangement will balance. One student sets up the board, holding the end so it is not free to swing; a second is asked to predict whether or not the board will balance. If the answer is "No," and the prediction is right, the first student can ask "What do I have to do to make it balance?"

This game is also worth playing with larger groups of students using chalkboard diagrams. One group puts a diagram on the board: Will this balance? The other students must decide as a group whether or not it will. This can generate good arguments about balancing principles and, also important, about representational systems. Did the first group represent on the chalkboard the situation they intended? Does everyone agree about what the diagram means?

There are many ways of analyzing a balance situation, ranging from a visual or fingertip feel for what does and does not balance to a careful calculation of the turning forces involved. The games just described encourage students to look in new ways at balancing situations. Symmetrical patterns of weights are quickly recognized by children as balancing situations. In addition, there is the symmetry of the balance-preserving moves that can be made with pairs of weights. This concept of symmetrical moves provides a link between symmetrical patterns and all those other arrangements that still balance but do not have pattern symmetry (that is, do not have the same pattern mirrored on both sides).

Starting with a balanced *asymmetrical* pattern, try to rearrange the weights into a symmetrical pattern, using only symmetrical moves. You should be able to do this in every case. Here is an example:

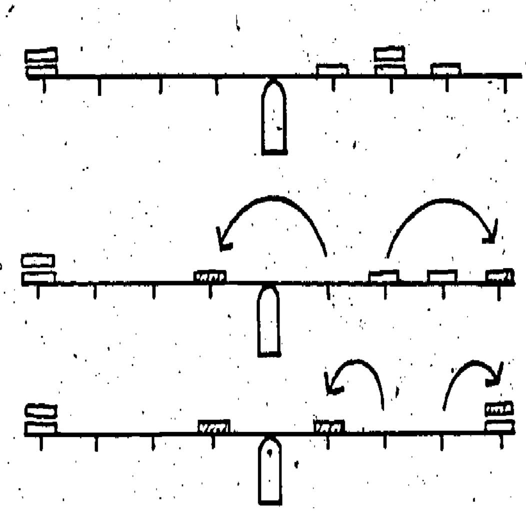
Does this balance?



Yes. (The one over the fulcrum doesn't affect the balance.)

Here is another:

Does this balance?



Yes.

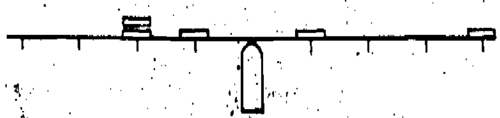
Just as a check, start with an asymmetrical case that does *not* balance, and try to transform it by symmetrical moves into a symmetrical arrangement that *does* balance. You won't succeed.

Often, children will use fractions in talking about the balance board: two washers out 1/2 as far will balance a single washer. This is a useful way of thinking about cases in which the numbers are fairly simple: 1 weight versus 2 weights, 1 vs. 3, 1 vs. 4, and even 2 vs. 3. This fractional view can be combined with the previous appeal to symmetry. For instance:

Does this balance?

Yes. A symmetrical move gives you two weights, 1/2 as far from the fulcrum.

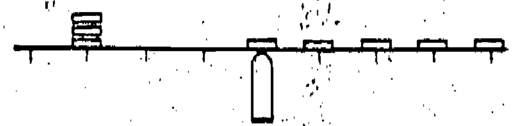
In working with balancing materials, use of one size of washer simplifies the investigation of underlying relationships. When equal-sized weights are used, both the symmetrical move and the fractional approaches can be described by a distance relationship that many children discover. Knowing this relationship is relatively unimportant; discovering the underlying regularity is exciting and worthwhile. Do not deprive your students of the chance to wrestle with this by themselves, even if you are tempted to share with everyone its discovery by one or a few students. The rule can best be explained by examples:



This balances, as seen by noting (1) the symmetry of the two single, inner washers and (2) the two washers halfway out vs. the third single one all the way out. Or, alternatively, on the left we have (1 washer @ 1) and (2 washers @ 2) while on the right (1 washer @ 1) and (1 washer @ 4)

$$(1 + 2 + 2) \text{ vs. } (1 + 4).$$

Will this balance? (Predict before you read on.)



It is not easy to tell just by looking, although you can transform the right side to give 3 @ 3 and then note whether the two sides are symmetrical. Compare distances in the preceding arrangement:

$$3 + 3 + 3 \stackrel{?}{=} 0 + 1 + 2 + 3 + 4$$
$$9 \neq 10$$

The arrangement will not balance; in fact, you can tell which way it will tilt. Did you predict correctly? Now go back and look again at the two problems on page 23 at the beginning of this section.*

*The section on off-center boards (see page 33) discusses how to predict when the board is supported off-center.

Problems Without Solutions

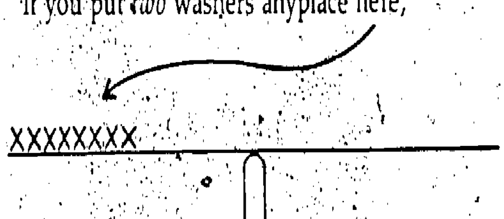
Problem Cards B1–B5

In one trial class, children were asked to make up problems for their classmates to do. One boy, with a twinkle in his eye, posed this one: "Balance the board in the middle, put two washers at one end, and balance the board with *one* on the other side."

Several children tried it and couldn't do it. Then there was general agreement that it couldn't be done ("unless," as one child said, "you move the board on the fulcrum").

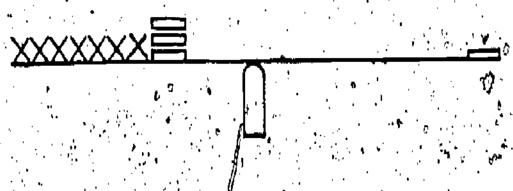
Could you put the two any place else and *still* not be able to balance the board with one? Yes, any place from the end to the half-way mark. At the halfway point the pair can be balanced with one at the end on the other side. As you move the pair further in toward the center, you can keep the board balanced by moving the single washer in toward the center, too, but twice as fast.

If you put *two* washers anyplace here,



. . . you can't balance the board with *one* on this side.

A similar problem is this: where can you put a pile of three washers on the board and not be able to balance it with one? Thinking about problems which can't be solved can lead children toward a better understanding of balancing, for knowing what can't balance also tells you something about what *can* balance. For example, here's one way to think about this problem. In order for the board to balance, the single washer must surely be out further than the pile of three. But the farthest out the single washer can be is at the end of the board, and then the three washers must be a third of the way out on the other side for the board to balance.



Thus, the outer two-thirds of that side is forbidden territory for the pile of three.

Any problem can lead a class on to unsuspected and interesting trails; perhaps a problem without a solution has a particular quality of stretching children's imaginations. See Appendix A for a description, written by the teacher, of two exciting classes with fourth graders, which evolved from a problem like the ones here.

What's Balancing What?

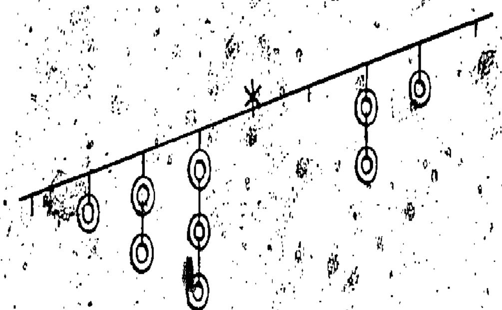
Problem Cards L1–L10

Sometimes a child will hang a complicated arrangement of washers on the board expecting it to balance, only to find it does not. "Why not?" he asks. The answer is not always simple to figure out, especially if there are many washers on the board.

To find out what is wrong, it is helpful first to think in terms of what is right.

The process here is one of mentally undoing or "stripping" the board by removing washers in groups that do balance each other until the offending washer or group of washers is found, and then deciding what to do to remedy the error.

Here is a problem as an example:

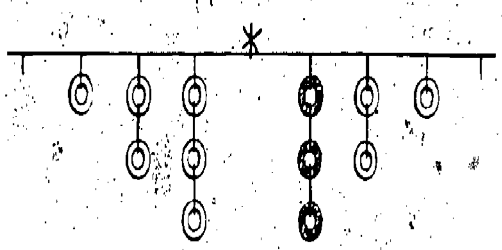


What's balancing what? Equal numbers of washers at equal distances balance as follows:

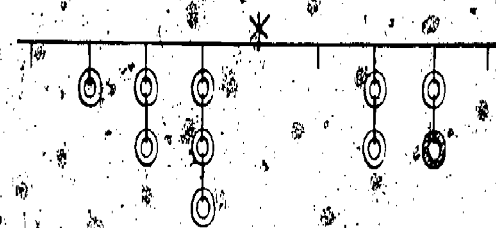
The single washers, each out 3/4, balance each other. The pairs of washers halfway out balance each other.

What's left? The three washers out 1/4 on the left aren't balanced by anything. Now you can think of ways to balance this board:

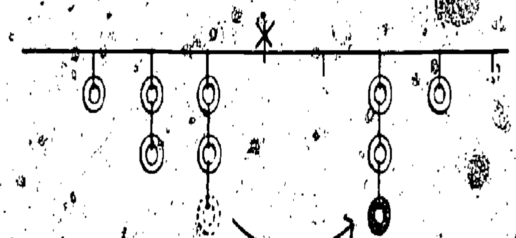
Add one or more washers, for instance, a pile of three, 1/4 of the way out on the right . . .



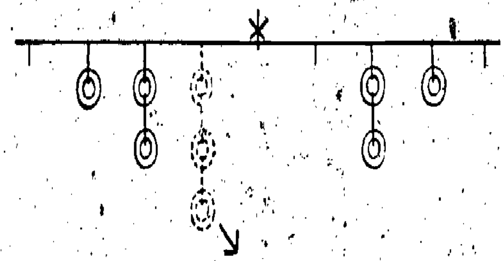
. . . or one on the right at 3/4 . . .



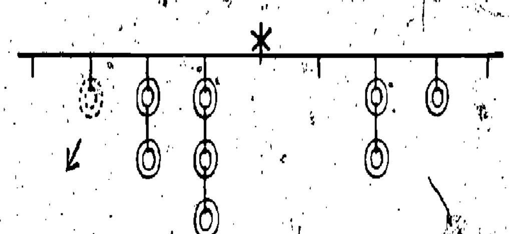
. . . or *move* one of the washers from the pile of three on the left across the fulcrum to halfway on the right . . .



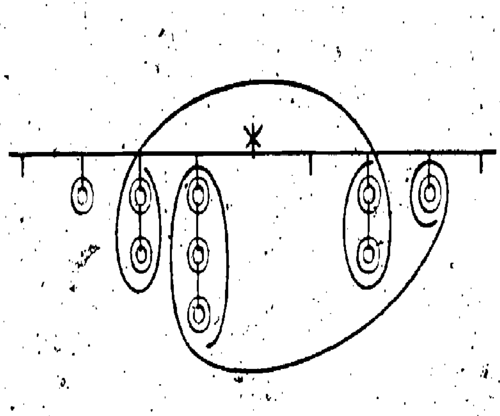
. . . or *take away* something: the whole pile of three . . .



. . . or the left washer at 3/4.

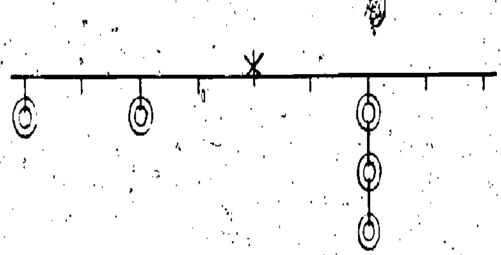


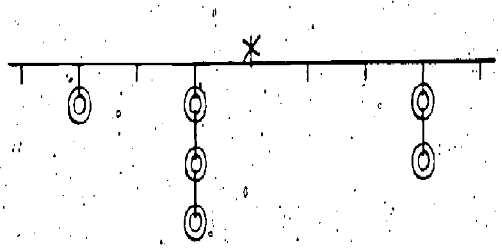
Note that with different pairings of "what's balancing what," the removal of the single washer becomes the obvious solution:



Up to this point, "What's balancing what?" has been discussed as a trouble-shooting technique. Even more than this, it is an important way of thinking about balanced situations, since it leads children to think in terms of the fact that two against one in the proper positions is as much "balanced" as a mirror-image pairing of washers.

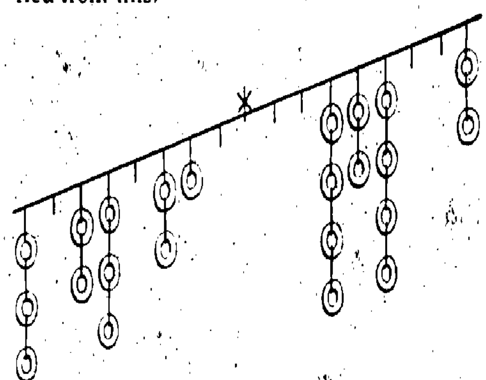
Here are more "stripping" problems:



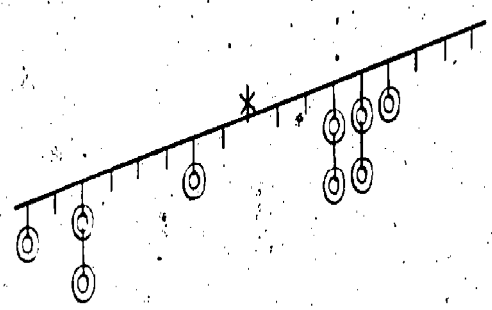


With a board that is *unbalanced*, removing groups of washers that balance each other does not alter the unbalance of the board.

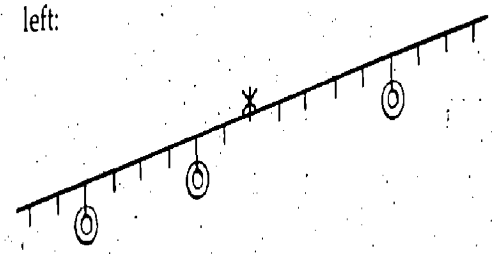
In analyzing complicated arrangements, it is often helpful to remove washers that balance each other in order to get to a simpler arrangement. For instance, this complicated board, which is not balanced, can be simplified from this:



. . . to this:



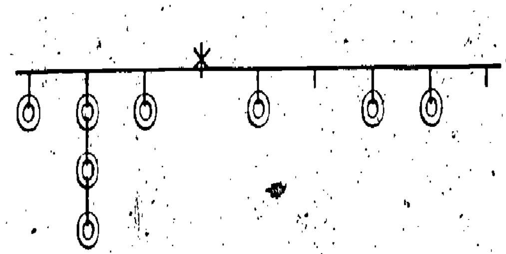
You simply remove from both sides of the board a pair of washers that match each other in distance from the fulcrum, and continue to do this as long as you can. After you have stripped the board further by taking off combinations where one washer on the left balances two on the right, you will have this left:



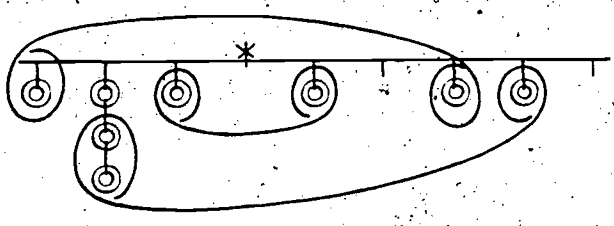
No wonder the board didn't balance.

Initially, puzzles like these can be attacked experimentally, even by trial and error. At some point, however, it can be fun to ask children to mark down on a diagram where they would add, take away, or move a washer without upsetting the unbalance—or the balance—of the board. Then, after many children have made their theoretical predictions, check with the board.

Another interesting puzzle involves off-center balancing. In these arrangements, some washers are balancing each other and some are balancing the board. Which weights are balancing the board? What's balancing what in this setup?

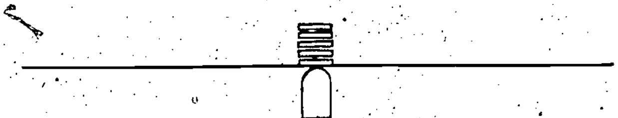


Well, first see which washers are balancing each other.



That leaves a single washer on the left which is not balanced by another washer; it must be balancing the off-center board.

Here's another problem: starting with a balanced board and five washers in the middle of the board (washers in the middle are like no washers at all in the sense that they don't affect the balance one way or the other),



can you:

1. move the washers two at a time?
2. keep the board balanced after each move?
3. end up with two in a pile on one end versus three in a pile on the other side?

What is the minimum number of moves you need to do this? Can you reverse the piles (so that the pile of three is on the left) in fewer than twice as many moves?

This type of question can be applied to any arrangement of washers. Either the nail fulcrum or block fulcrum can be used in working out the solution. The advantage in using the block fulcrum is the greater ease with which the moves can be made. Once children have discovered the two kinds of symmetrical moves that maintain the balance of the board, they sometimes remove the fulcrum completely and work with the pegboard flat on a table top, returning the board to the fulcrum only to check an answer.

Some further questions to investigate:

1. Weight one end of the board with modeling clay. Find the balance point, and then try again some of these adding and moving questions. What's balancing what? What's balancing the washers? What's balancing the clay? Is the problem any different? Are there any questions you solved with a symmetrical board that you cannot solve with this asymmetrical board?

2. If you use some heavy washers and some light ones on the board at the same time, what questions can you ask? How do the possible moves differ from ones involving washers all of the same weight?

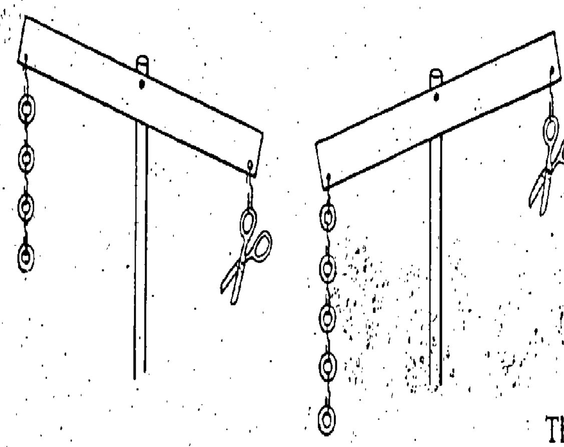
Weighing — Problem Cards E1–E8

One important use to which a balance can be put is weighing things. Indeed, one of the first things some children do with the balancing equipment is to find out how many paper clips a washer weighs by putting the nail in the center, the washer on one end hole, and the paper clips on the other end hole. The visual symmetry of the balanced board is probably what convinces children that the paper clips weigh the same as the washer.

The weight of a small washer in paper clips leads to an interesting problem. If you also use a board to find out how many small washers a large washer weighs, then by using some arithmetic, you can find out how many paper clips a large washer weighs. To check your answer, of course, you should actually balance a large washer against paper clips.

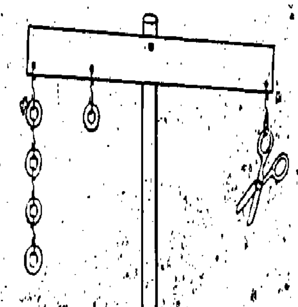
The balance can be used to weigh many objects around the classroom. In a fifth grade one girl went around finding the weight of each student's shoes. She kept a chart of the shoe weights.

Scissors are a convenient item to try weighing.



Four washers may be too few; five too many. The process of weighing – indeed, of making any measurement – involves narrowing the answer down to somewhere between two values. Still, we can get closer than between four and five. A perfectly good thing to do is to guess – say 4 3/4 washers. An alternative is to use paper clips as fractional weights. Twenty paper clips are about right. If you have already weighed washers in terms of paper clips, you know that 32 paper clips are equivalent to 1 washer. This means 20 paper clips would be 20/32 or 5/8 of a washer. The scissors, then, weigh about 4 5/8 washers, pretty close to the guess of 4 3/4.

Another approach is to place the fifth washer only partway out on the board.



The best hole for this washer seems to be ten spaces out from the nail. For example, 10/16 (or 5/8) of the way out on the board is the same as 5/8 of a washer at the end of the board. Again this gives a weight of 4 5/8 for the scissors.

Still another way to get fractional washers is to use an off-center balance. If you have the board in the ninth hole from one end, using modeling clay on the short end to balance the board, then there will be 8 holes on one side of the fulcrum and 24 on the other. Should you put the scissors on the long end or the short end of the board in order to get the most sensitive weighing? Where should you place the fulcrum so that every washer added on one end will measure 1/7 of a washer's weight on the other end?*

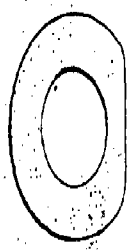
Try weighing a piece of paper. Spear it on a paper clip in order to hang it on the board. How many paper clips does the paper weigh? How many washers?...

If you crumpled the paper up into a ball, would it still weigh the same? What if you cut the paper into strips?

A somewhat related problem was worked on by one sixth grader. He tried balancing a washer, hanging at the end of a long string and then balanced it again with the string bunched up in a ball. He was sure there would be a difference. He reasoned that when the washer was closer to the earth, it would be pulled down harder. Therefore, raising the washer up and hanging it on the beam should make that side of the beam tilt up slightly.

In fact, he and his classmates couldn't see any change. His reasoning was right in a sense, but the effect he expected was much too small to be seen in the experiment he was doing.

Occasionally a washer is missing a piece and looks like this:



It was probably stamped out too close to the edge of the sheet of metal. One child took the opportunity to find out about how much was missing by weighing this washer in terms of paper clips and comparing his answer to the weight of a whole washer.

Weighing things in terms of washers or paper clips may be a little of a puzzle to some students at first. They may feel that weights should be in pounds and ounces—or in kilograms and grams in some parts of the world. Why use washers?

*This is an example of a balancing situation in which the two sides of the board balance each other, but the weights on one side do not weigh the same as the weights on the other. In many balanced situations, the two sides of the balance do not weigh the same.

If this question comes up in class, the following may help in any ensuing discussion.

As units of weight, washers will do about everything for your purposes that pounds or ounces will. Originally, someone decided to call the weight of a certain chunk of metal a pound, made replicas of it (probably using a balance), and persuaded other people to use the pound as their unit of weight. Then the weights of all other things were referred to as so many pounds. It didn't matter what was used as the original standard; it had to be of a convenient size, easily reproducible (so a great many people could use it), and relatively permanent. Certainly, other choices for the standard of weight are possible and, historically, have been made. The kilogram is the most familiar one. This is defined as the weight of a special chunk of metal kept in Paris, France. Both the pound and the kilogram are about the same size, in the sense that they are convenient for expressing the weights of everyday objects, objects which man can easily lift. For much heavier or lighter things other units are useful, such as the ton and the ounce, which are both defined in terms of the pound—as a multiple or a fraction of it—so that no new chunk of metal is needed.

So the "washer" as a unit of weight is perfectly respectable for your class. The washers are of a convenient size and plentiful, and they're relatively permanent. There are some disadvantages, though. The washers of one size aren't all quite of the same weight, and, outside of your classroom, the "washer" isn't a widely recognized unit. The man at the meat counter would probably give you a peculiar look if you asked him for 20 washers of hamburger. If you had washers with you, he could weigh the washer in terms of *his* units to find out how many pounds a washer weighs. Then he could calculate how many pounds 20 washers weigh and use his scale to weigh out that much meat.

Some children enjoy making a series of weights to weigh objects directly in ounces. By using an ounce weight, such as a fishing sinker or a square of baking chocolate, as a standard, they can make a set of clay lumps for weighing items of a pound or less to the nearest ounce, using an equal-arm balance. One solution is to make 16 individual ounce weights. This, of course, uses 16 ounces of clay and means making 16 different pieces.

An interesting challenge related to making that set of weights is to ask how *few* separate pieces are needed. If you use more than 16 ounces of clay, it is possible to make all weighings with only four separate clay lumps. If you restrict the amount of clay to 16 ounces, five are necessary. What is the weight of each of those lumps?

Off-center Boards

Problem Cards M1–M3

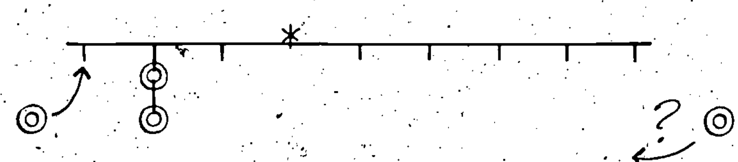
A centered balance board is simpler; an off-center balance board is more interesting. One reason for looking at off-center boards, aside from their intrinsic appeal, is that many of the relationships that work for centered boards still apply. To find a familiar relationship in an unfamiliar context—to see the sameness in apparent difference—can be exciting as well as instructive.

Children's first off-center balancings will probably be complex ones, achieved by trial and error: adding washers, shifting the fulcrum, or both.

For those who want to find some rules, here are two off-center problems to investigate:

1. Starting with an off-center board already balanced, how can you add more washers and still preserve the balance?
2. How do you balance the board with the nail off-center in the first place?

Here is a particularly interesting case of question 1:



Where do you hang another washer to rebalance the board? Many children will first try the end hole on the long side, which would have worked had the nail been in the center of the board. But it's not where the ends of the board are that matters; it's where the nail is, and this problem at any stage of a child's work can help focus his attention on the importance of measuring distances from the nail.

Try balancing other combinations of washers on this board: 1 vs. 2, 1 vs. 3, 2 vs. 3, piles vs. spread-out. There's a marvelous similarity between these and the same problems done on the board with the nail in the center. In both cases the board is balanced to begin with, in one because the nail is in the center, in the other because some washers have been added to make it balance. It doesn't matter how long the board is, what it's made of, or how heavy the weights are (provided they are all equal). In each case the same moves keep it balanced.

How you balance an off-center board in the first place is a difficult problem. One approach is to gather information on specific cases. Then, after you have data, you can look for underlying regularities that may reveal a general relationship.

With the nail three spaces off-center, various ways of balancing the board were found, always adding washers only to the short side:

1 washer out 12 spaces from the nail
2 @ 6
1 @ 9 and 1 @ 3
1 @ 7 and 1 @ 5
3 @ 4
1 @ 5, 1 @ 4, and 1 @ 3
4 @ 3

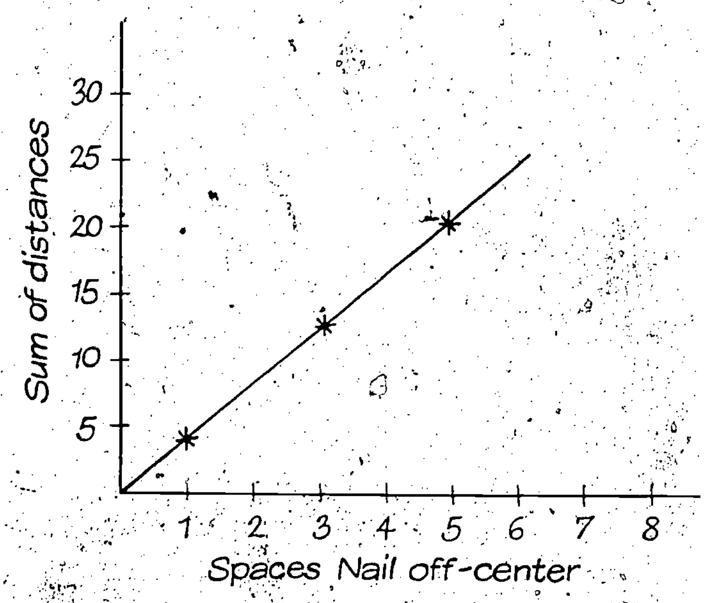
In each of these cases, the sum of the distances of the washers equals 12.

Before a general relationship becomes evident, you need results for other cases.

Spaces nail is off-center	Sum of washer distances
3	12
1	4
5	20
4	?

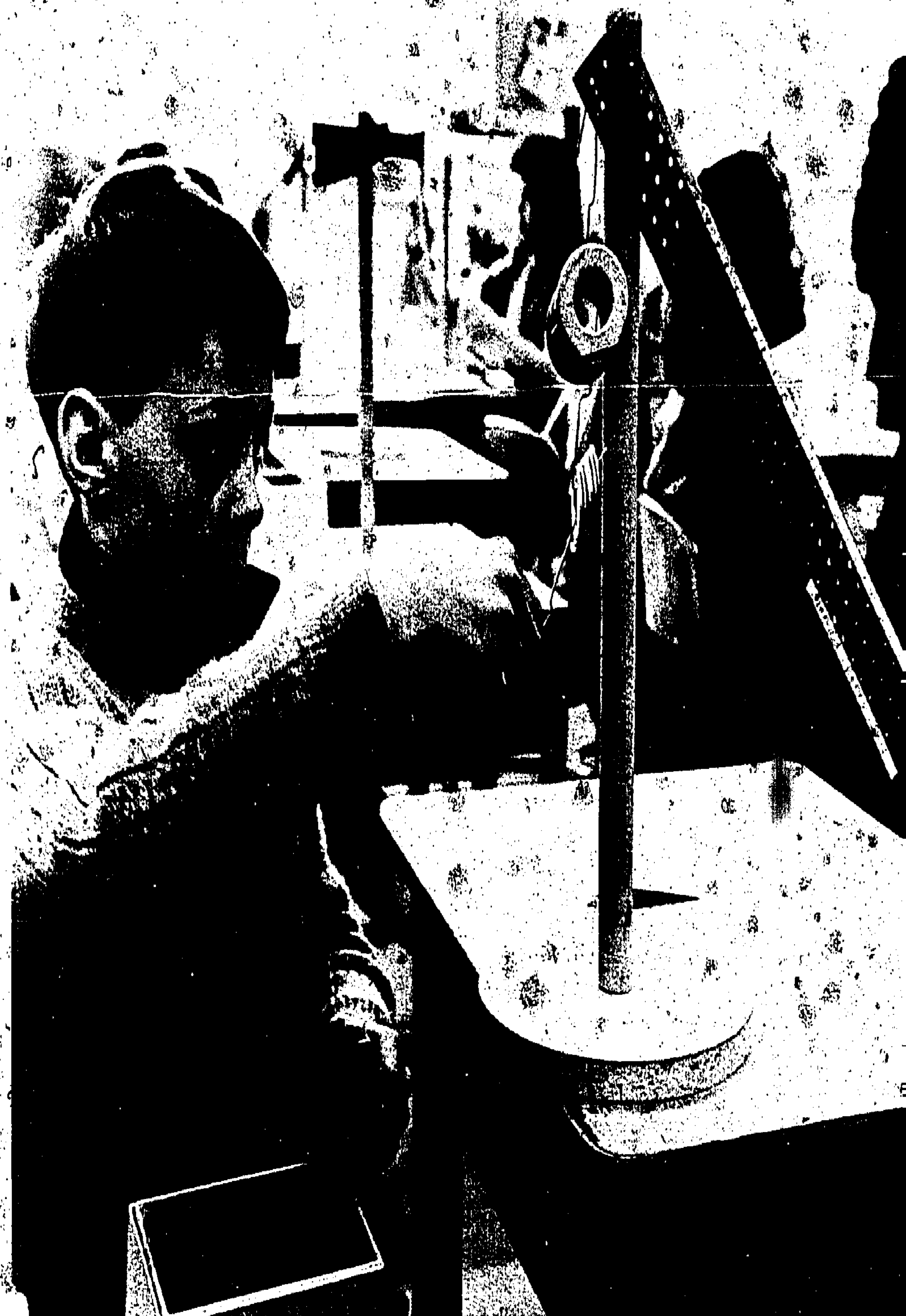
Can you predict for additional off-center cases?

Is there a simple relationship between spaces off-center and sum of distances? A graph can be helpful:



What you see in both the table of data and the graph of the same data is that the sum of the washer spaces is always four times the nail off-center spaces. This is a nice thing to find, but it is not crucial to an understanding of balance boards, and you should suggest the problem gingerly.

The end-hole problem In one sixth grade, two boys were trying to find how many washers they needed to balance the board with the nail in the end hole. The teacher had been in another part of the room but became involved when an argument broke out: the boys had used all their washers and were trying to get more from a neighboring group. With twelve washers, the board was far from balanced. With twenty more washers borrowed from other groups, the board was still not balanced. By now, the washers, hung in groups on shower-curtain rings, were dragging on the table, and the boys put a pile of books under the stand to elevate the board. The teacher called everyone's attention to this problem: how many more washers do you think they'll need to balance the board? Guesses ranged from 12 to hundreds. More washers were collected and strung on a length of cord. With several hundred washers—over 30 pounds—the board was almost horizontal . . . but not quite. At this point the teacher took hold of the cord and pulled down as hard as he could. Was the board horizontal now? No, not quite. If you sighted the board against the cinder blocks of the classroom wall, the board and the horizontal seams were not quite parallel.

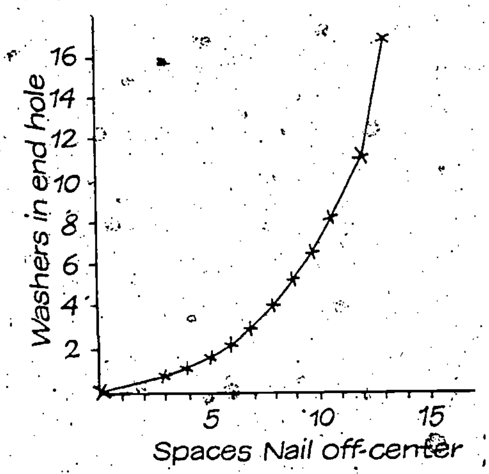


"What do you think?" asked the teacher.
"More washers," said the boys.

They were wrong; more washers were never going to make that board horizontal. Two alternative investigations that should enable a student to resolve this problem are described below.

One approach focuses attention on putting washers in the end hole, but with the nail in some other hole.

Record the number of washers needed in the end hole to balance the board for various positions of the nail. Here is a graph of the data from one such table:



"Sixteen spaces off-center" represents the case in which the nail is in the end hole of the board. The curve will never quite reach the vertical line drawn at 16, that is, there is no possible number of washers that will balance the board when the nail is 16 spaces off-center.

Another approach focuses attention upon hanging washers directly under the nail. Suppose the board is centered and balanced with 1 @ 3 + 1 @ 5 = 1 @ 2 + 2 @ 3. What will be the effect of hanging additional washers directly under the nail? Next try adding washers under the nail, with a balanced off-center board. One more step: put the nail in the end hole, hold the board in a horizontal position, and add washers beneath the nail. But first, guess what you will feel in your hand: more push? . . . less push?

You can understand all this mathematically. We have already seen that one washer will balance three out 1/2 as far; one out 6 will balance three out 2. The distances add up here: $6 = 2 + 2 + 2$, which can be written as $6 = 6 \times 1$, or 12 out 1/2: $6 = 12 \times 1/2$. Putting washers directly under the nail, however, is zero distance away, and zero times anything still equals only zero. This is why holding the board horizontal when the nail is in the end hole and adding weights to the end hole has no effect. This is why the graph goes towards infinity on the vertical axis. This is why the boys were never able to get their board horizontal, even with 30 pounds of washers.

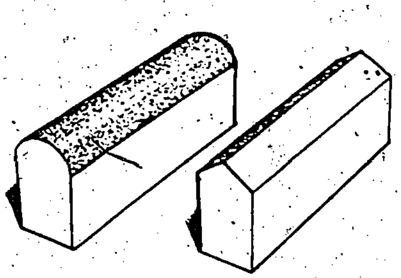
Why is it, then, that adding washers to the end hole, with the nail in the end hole, had any effect at all? Check back with your own board, or refer to the picture on page 35. When the board is not horizontal, are the washers hanging directly under the nail?

Extensions

Problem Cards S1–S7

What Makes the Balance Work?

This section is intended primarily as source material for you, although an able and interested student might go on to try some of the investigations described here. While reading this section, you should have two balance setups at your side. You will also need the plastic strip and the block fulcrum with a knife-edge top that are included in the *Teacher's Kit*.



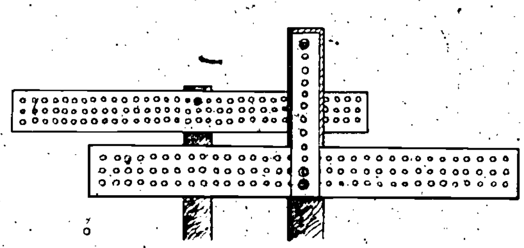
A balance seems like a simple gadget. To make one, all you need is a stick of some sort and a fulcrum. The pegboard strip balances easily when the nail is placed in the upper middle hole; the pegboard also balances easily across the rounded block fulcrum. The top of the block fulcrum is cut in a circular arc.

Why go to the trouble of cutting the curve? Here's a way to find out:

Included in the *Teacher's Kit* is a block fulcrum cut with a triangular top, like the roof of a house. Try balancing a pegboard strip on this fulcrum. Will it balance? It seems as if it should be simple, but it isn't. No matter how carefully you try, the balance is always unstable. Even if you get it balanced, a slight nudge will tip it over.

To see why the board will not balance on this knife-edge fulcrum, look at a different situation. You know that the pegboard balances easily on the nail fulcrum when the nail is in the top-row middle hole. Have you tried balancing the board with the nail in the middle-row middle hole? . . . in the bottom-row middle hole? When the nail is in the bottom row, the board does not behave like a balance.

If you attach the plastic strip to the pegboard, as shown below, you can try cases in which all of the board is beneath the point of suspension. By setting up a second board with the nail in the top-row middle hole, you can compare the behavior of the two boards. Try various holes in the plastic strip. What happens when you add a single washer to the end of each board? How many washers are needed to make each board tilt the same amount?



The important property of the board that is involved in these matters of stability is its "center of gravity," a term whose meaning should become clear to you from its context if you are not already familiar with it. For the pegboard, the center of gravity is at its geometric center: in the middle hole of the middle row.

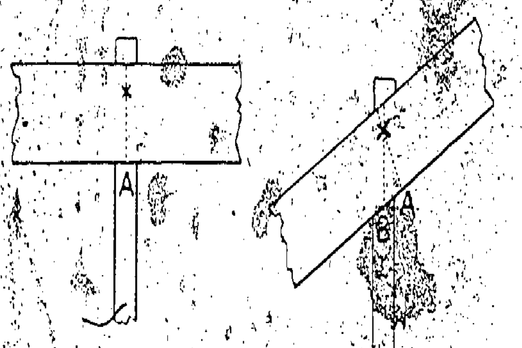
The plastic strip allows you to see what happens as the center of gravity is moved further and further below the fulcrum. A board will balance when its center of gravity is below the fulcrum; and the further it is below the fulcrum, the more stable the balance will be. You saw that this was true when you tried different holes of the plastic strip. This is why the board would not balance on the wood knife-edge: with the center of gravity of the board above the fulcrum, the board is unstable and will always tip one way or the other.

Yet one look at the curved-block fulcrum seems to throw this whole theory into a cocked hat. The board balances nicely on top of the curved fulcrum, even though the center of gravity of the board is above the fulcrum. You might look carefully at the board balanced on a curved fulcrum and see if you can determine what is making the board stable and compensating for the high center of gravity.

horizontal—which is hard to do with the flat fulcrum—the opposite would occur: more board would stick out to the right of the point of contact with the fulcrum, and the unbalanced force would again move the board back toward the horizontal. All stable balances exhibit this rocking action caused by the setting up of a *restoring force*, which opposes any tilting of the beam.

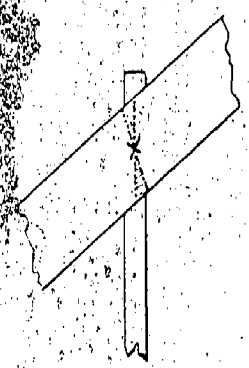
Now go back and look again at the board on a curved fulcrum. Notice that as the board tilts there is a rolling effect which causes the point of contact between board and fulcrum to shift. With more of the board out one side, a restoring force results which causes the board to rock back. This effect more than compensates for the instability caused by the high center of gravity. With a knife-edge fulcrum there is no such compensating effect, the point of contact remains fixed, and the high center of gravity makes the board unstable.

When the pegboard beam is hanging on a nail fulcrum, there is no such rolling shift as the board tilts, yet there is still a restoring force. How does this happen? Well, in a sense, the same effect takes place; look at the diagrams which follow.



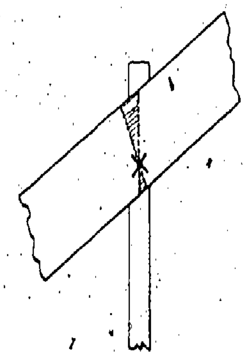
As you know, adding washers directly beneath the nail has no turning effect on the board. The point directly beneath the nail is at A in the horizontal case, but when you tilt the board, A is no longer directly beneath the nail, and you would have to hang the washer at B in order for it to have *no* effect. The vertical line from the nail marks the zero position. When the board is horizontal, the pegboard is equally distributed on either side of this vertical line; but when you tilt the board, you shift more of the board out the other side. There is the restoring force.

All this discussion has assumed that the nail was in the middle hole of the top row. What if you place it in the middle row?



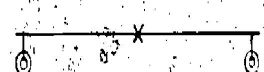
Notice that the shift of extra board to the right beneath the nail is exactly compensated for by a shift of board above the nail to the left. This is why the board stays in any position in which you put it—there is never any restoring force.

Now look at the case in which the nail is in the bottom row. You know that this makes the board unstable; once it starts to tilt, it flips all the way over.



No wonder; as the board tilts, it shifts extra board above the nail on the same side of the vertical as the tilt. In other words, a tilt produces a *reinforcing force* in this case instead of a restoring force.

It is interesting to consider abstract drawings, such as the following:



If you were to build balances that looked exactly like these diagrams, they would never balance. In each case the abstract drawing leaves out the crucial feature that produces a restoring force when you tilt the beam: the curve of the fulcrum in the first case, and the width of the beam in the second case.

74

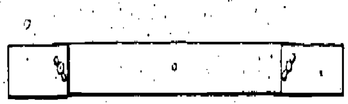
Challenging Problems

These problems are intended for those students who are particularly apt and are ready to work on more advanced questions on their own, or for students who are so captivated that they want to continue working with balances above and beyond the rest of the class. Each problem is reproduced on a student card. Included here, in addition to the problems, is an analysis for the teacher. These analyses are somewhat briefer and presume more understanding than do those elsewhere in this unit.

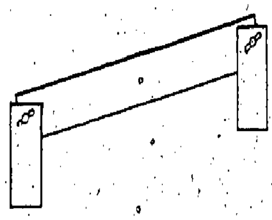
Problem #1:

Bolt a quarter-length strip to each end of a beam, and hang it with the nail in the middle hole. Decide on your answer to each of these questions before trying it on the board.

a. Clamp the arms as extensions of the board. Spin the board. Where will it stop?

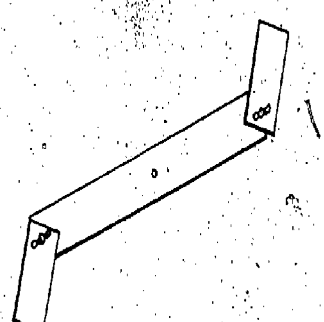


b. Leave the arms free. Spin the board. Where will it stop?



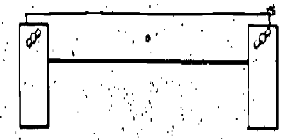
c. Clamp the arms where they stopped for b. Spin the board. Where will it stop?

d. Clamp the arms. Spin the board. Where will it stop?

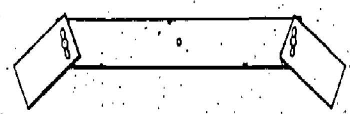


e. Which one will rock faster,

this?



. . . or this?



Analysis:

This series of questions should bring some surprises. They are valuable in that they present someone with a situation in which his initial intuitions are in error, so that he must deal with his own mistakes and inconsistencies. A careful analysis is similar to that given for Problem #5 (page 43).

Problem #2: (Problem #1 is a prerequisite for this problem.)

Bolt a quarter-length strip at each end of a beam, and hang the beam with the nail in the *bottom*-row middle hole. For each of the following questions, decide what you think will happen *before* you try it.

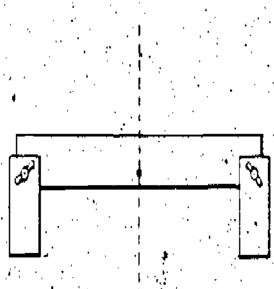
a. If the strips are free to swing, what will the beam do?
b. If the strips are bolted tightly at right angles to the beam, what will the beam do?
c. Keeping the strips tightly bolted, how far do you have to rotate the beam about the nail before it will flop over instead of rocking back? What if the strips are free to swing?

Try asking someone else these questions while you handle the board; if they can't "feel" the answer, they have to think all the harder.

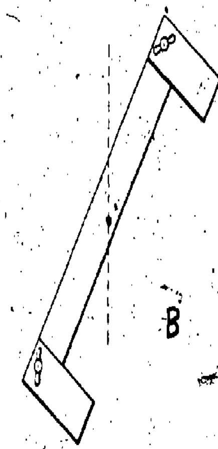
Analysis:

(Questions a and b are answered by the Analysis of Problem #5, page 43.)

Question c:

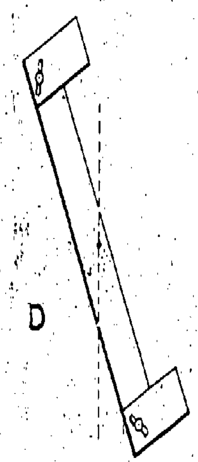


A

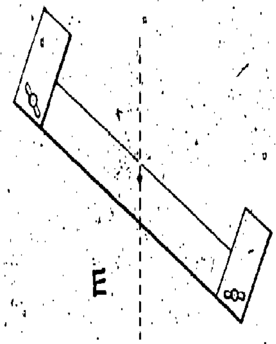


B

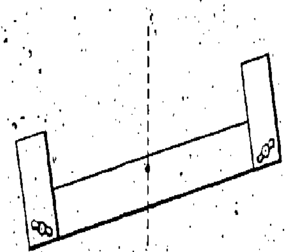
C



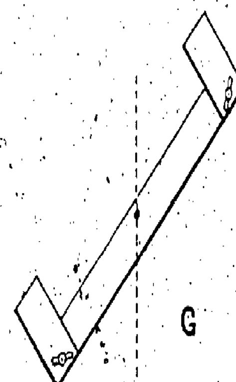
D



E



F



G

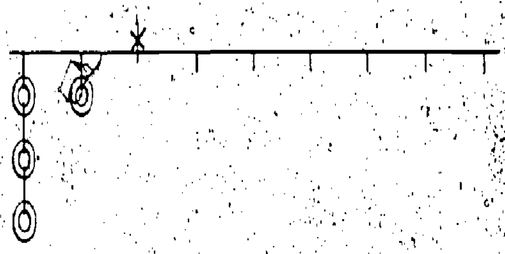
In diagrams A through G, the important feature each time is the comparison of the distances from the vertical line to the center of each strip. As the beam is turned counterclockwise, a restoring force is set up in every case except the last two. In F and G, a reinforcing force is established, and the beam will flop on over to position A.

Problem #3:

Using only one pegboard beam, can you find the weight in washers of that beam?

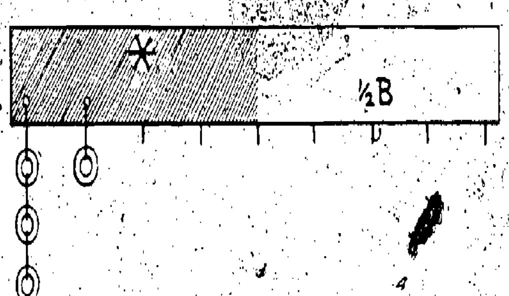
Of course you can always check yourself by actually weighing a beam on another balance. Don't do this until you have figured out how to find the weight using only one beam. (Some students have worked for several days on this problem.) Can you explain how you found your answer?

Analysis:



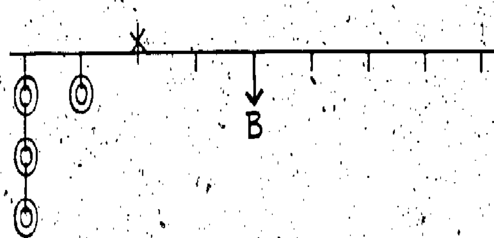
When the fulcrum is off-center, as above, the weight of the board becomes a factor. Two ways of determining the weight of the board, explained in terms of the example above, are as follows:

a. Symmetry: the two units of beam on either side of the fulcrum balance each other.



The four washers must be balancing the remainder of the board: 1 @ 1 + 3 @ 2 = 1/2B @ 4. Here 1/2B represents the weight of the unshaded portion of the beam, acting at the center of this section.

b. A more quantitative approach:



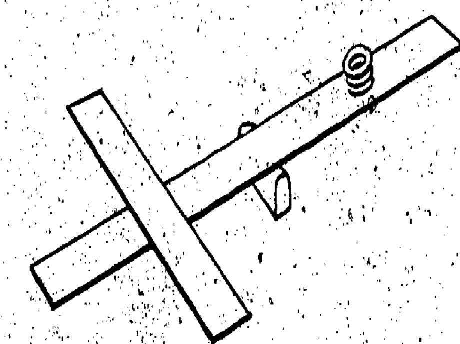
The weight of the whole beam, B, can be thought of as concentrated at its center.

This weight, out two units from the fulcrum, is balanced by the washers on the other side of the fulcrum. $3 \times 2 + 1 \times 1 = B \times 2$.

Problem #4.

You will need a half-length strip of pegboard in addition to a regular-length board, a block fulcrum, and some washers.

a. Balance the board, as shown, with a half-length strip across the beam halfway out.



Predict what will happen if you pivot the strip so that it lies along the board. Try it.

b. Predict what will happen if you start with the strip near the end of the board . . . with the strip in near the fulcrum. Were you right?

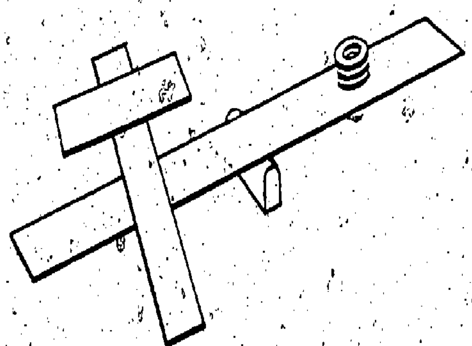
c. Try pivoting experiments with the above arrangements.

d. Try each of these experiments with a bolt in the beam for the strip to pivot on.

Analysis:

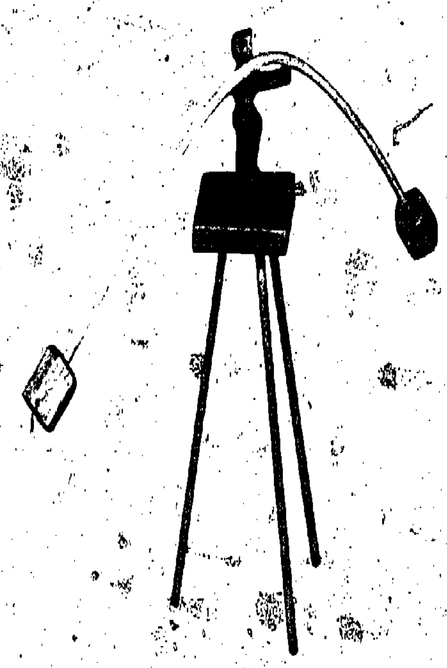
Many people intuitively feel that pivoting the half-length strip will make a difference when the strip hangs over the end or overlaps the fulcrum.

The situation involving a half-length strip with a quarter-length strip across each end is basically the same as the situation with the half-length strip alone—in both cases, there is symmetry about the center of the strip. With a quarter-length strip across only one end of the half-length strip, however, this simple symmetry no longer applies. If you pivot about the center of the half-length strip, the balance board will not remain in balance.



Placing the strip on a bolt does not change the results of these pivoting experiments. The strip acts as if all its weight were concentrated at the center, whether the bolt is there or not.

Problem #5:

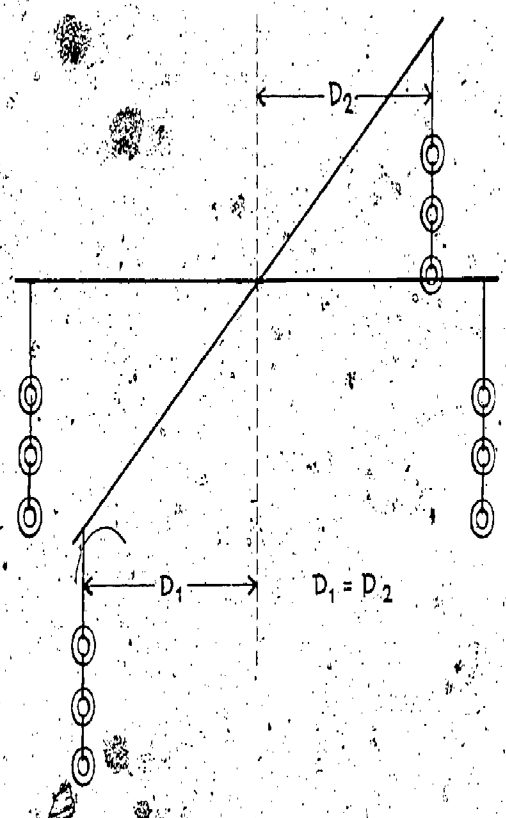


This toy man balances because the weight on his hands is down so low.

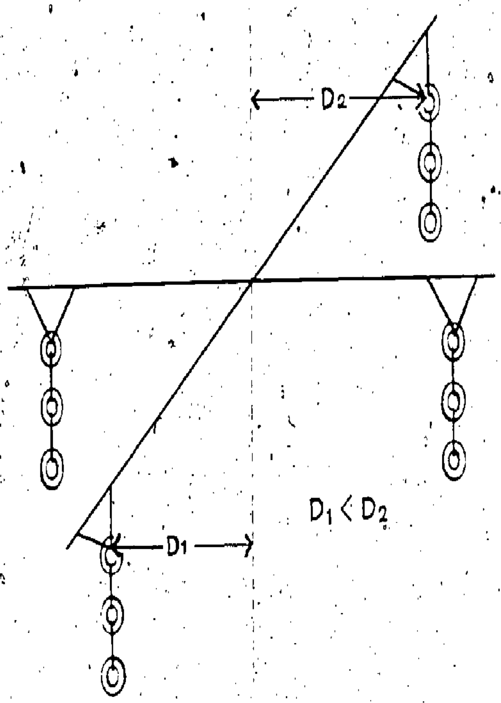
You know that the pegboard is unstable if you put the nail in the middle hole of the bottom row. The balance man suggests that the pegboard would be stable in that situation if you hung washers down low on long chains of paper clips from each side. Try this; it does not help. Yet it is possible to make the balance stable even though the nail is in the bottom row. Can you do it?

Analysis:

Imagine a vertical line running through the fulcrum. When the board is tilted, the distance from any weights to the fulcrum is measured to that line rather than along the beam.



As you can see in the drawings above, when the weights are hung so that they can swing, the distances remain equal.



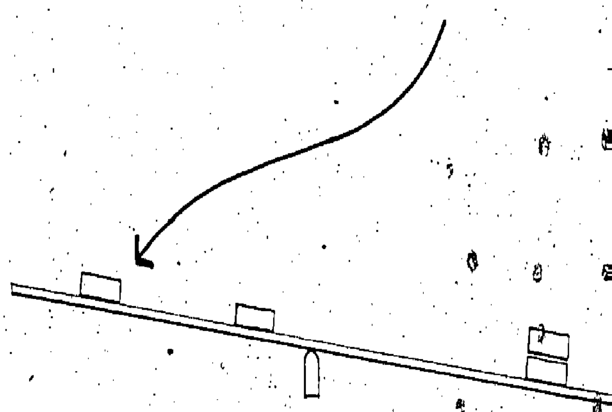
When the washers are held more rigidly, as by a yoke of interlocked paper clips, the distances become *unequal* when the board is tilted. This sets up a restoring force and makes the beam stable.

Appendix A

An Example of Unexpected Happenings

The following teacher's account is reprinted here for two reasons: it is an example of how work with balancing can lead into exciting, unplanned investigations, and the classroom diary style of the account provides a picture of one class as it actually was.

We were working with a four-foot-long balance board and blocks of equal weight, and I had set a problem which could not be solved. "Who can balance the board by moving this block?" I asked.



Several hands went up, and one boy came up and moved the block out to the end of the board. It still did not balance. I put the block back to its original position. Several more had ideas, but no one could make the board balance by moving the extreme left block.

"You need a couple of more feet of board out there, don't you?" I asked one boy who was trying to balance the block half off the end of the board. The class laughed, but still more put their hands up insisting they had a way. A boy came up and, with a great smile of triumph, stood the block up on end.

"Why did you do that?" I asked.

"To balance the board," was the reply, and he proceeded to move the upright block along the board.

"Do you think it weighs more standing up?" I asked.

"Yes," he said.

"I think I said 'Oh.'" While I collected myself, he continued to move the upright block around the board.

"Do you think *you* weigh more standing up or lying down?" was my next question.

"Standing up," he replied.

"Oh, that's interesting," and I paused a moment. "Why?"

"Because you press down harder when you are standing up."

"I see. Well, you know that would be an interesting experiment. When you go home you ought to go into the bathroom and weigh yourself standing up and then lying down, and see if you really do weigh more standing up." The class laughed as they imagined what it might be like trying to lie across a bathroom scale. The board still would not balance though, and we had to agree that standing the block on end had not made it weigh enough more. This class ended a few minutes later, after we had balanced the board by moving some of the other blocks.

That boy was right in a sense. Although your weight is the same, standing up or lying down, your total weight is concentrated in a small area—the soles of your feet—when you're standing up, and spread out over a big area when you're lying down. We decided to follow up this problem of standing up versus lying down. I came to the next class equipped with two bathroom scales and a plank. Everybody remembered the funny idea of lying down on a bathroom scale to weigh yourself. They could see what was coming. But before we actually weighed a child standing and then lying down, I asked what they expected.

"He'll weigh more standing up because he pressed down harder," said some.

Another child said he would weigh more lying down.

"Why?"

"Because when you're standing up your muscles help to hold you up."

"The scale will read the same whether he's standing up or lying down," said another boy.

"Why?"

"Because he still weighs the same, because he's still just the same person," he explained. Nobody else seemed to be influenced by this reasoning.

A second boy subscribed to the muscle theory.

"I can lift up my brother if he's lively and playing around, but if he's lying down asleep, I can't."

"I know what you mean," I said. "All that dead weight really makes it hard."

The problem was more fascinating than I had imagined. It was time to settle the issue. We weighed our stand-up-push-harder boy of the previous class while he was standing: 60 pounds. Then I placed the plank across the scale, and he stretched out on it: 70 pounds. "He weighs more lying down," I said. The class was not fooled. "The plank weighs extra," they pointed out.

"How much do you think the plank weighs?"

"Three pounds, maybe," volunteered one girl.

"Ten pounds."

"Nah, it doesn't weigh that much."

"Twelve pounds."

We weighed the plank alone: 10 pounds. We weighed our boy standing up on the plank: 70 pounds. I was pleased with how faithfully the bathroom scales were performing. The children, I sensed, were a little disappointed with the result; the world was not as imaginative as they had hoped. Now I brought out the second scale and laid the plank between the two. We would repeat the lying-down experiment, I explained and asked what they thought each scale would read.

"Seventy pounds each," was the most popular answer.

"Sixty-five pounds each," said one boy.

"Why only 65?"

"Well, I figure the board is held up by both scales, so that's only five pounds each."

"So each scale supports just half the plank," I reiterated, "and then when he lies down it will be 60 pounds plus only 5 pounds on each scale instead of 60 pounds plus 10 pounds as before." The boy nodded in agreement.

Another child suggested that one scale would read 35 and the other 58 pounds. When I asked why, he explained that the feet weigh more, so that scale would have the most weight. This bothered another boy, and he said the scales would read unequally in the opposite order because the head was heaviest. A third boy said each scale would read 50 pounds because the stomach was the heaviest part of the body.

The boy who had said correctly that a person would weigh the same standing or lying down again had a penetrating insight into the situation. "Each scale will read 35 pounds," he said.

Another jumped up, "No, 36 pounds each, because of the two extra blocks you are using to hold up the plank." They smiled in agreement.

We decided to try the experiment. We laid our stand-up-push-harder boy down on the plank, and two girls read the dials. We shifted the boy back and forth to different positions on the plank, writing on the blackboard the readings of the two scales each time.

Scale A	*Scale B*
29	42
33	38
50	23
39	33

"What is there that is the same about each set of readings?" I asked. "Do you see anything that doesn't change?"

Two hands went up eagerly. "Don't say your answer yet. Let's give everybody a chance to figure it out."

I went to the board and rewrote each pair vertically:

29	33	50	39
42	38	23	33

"Who sees something the same about each set of numbers?"

Hands began to pop up all over the room, accompanied by gasps as each child suddenly saw the underlying regularity. I pointed to one child, and a great chorus rose from around the room:

"They all add up to the same thing."

I was delighted that so many children had seen the relationship among the pairs of numbers for themselves, and that they had the good judgment to accept the sums—71, 71, 73, 72—as "all the same" under the circumstances. The small differences, caused by inaccuracies in the scales or in reading the dials, didn't keep the children from seeing the basic relationship.

In the class time left we tried the double-scale experiment with two other students as subjects. Now, we had the students read only one of the scales and then asked them to predict what the second scale would read. The children had discovered the basic law, and now they enjoyed seeing it work. Variations were eagerly suggested by the class.

"Try it with her standing up on the plank."

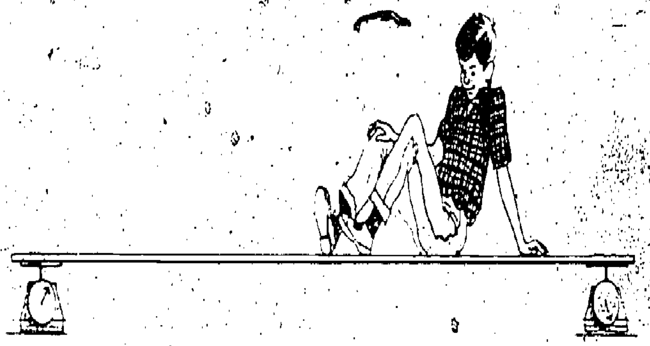
"Where should she stand to make both scales read the same?"

"What if she stands all the way over on one side of the plank?"

"What if he sits down? . . . kneels down?"

In the middle of all this hubbub, a light suddenly shone in one quiet girl's eyes. "It's just like the balance board," she said.

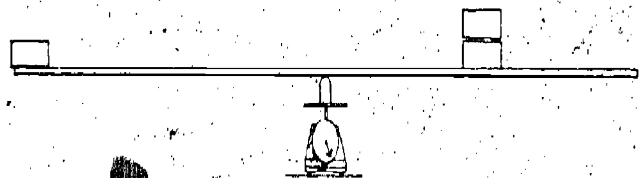
I wasn't sure it was just like a balance board, but some of the other children understood. "Yeah," they said, and late that night I also saw the connection: "Oh, yeah," I exclaimed to myself.



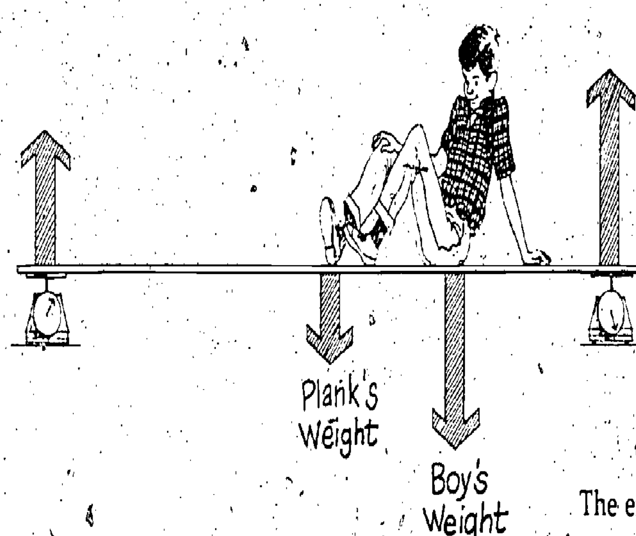
Think of it as an upside down balance board. The person on the plank is like the fulcrum upside down, and each scale pushing up on the plank is like a pile of blocks pushing down on the board.

When the plank is "balanced," the scale closest to the person pushes up the hardest, and when the board is balanced, the pile of blocks closest to the fulcrum pushes down the hardest—it must be the bigger, heavier pile. I suspect this is the similarity that the girl saw.

What about the discovery the children made in class—that the readings of the scales always add up to the total weight of the person plus the plank? The scales together push up just as hard as the person and plank are pushing down.



But, is the upward push of the fulcrum always equal to the weight of the board and blocks pushing down? It is, and placing the fulcrum on one of the bathroom scales will show this.



Does this work the same way for the balance board? The blocks and board are pushing down, and the fulcrum must be pushing up.

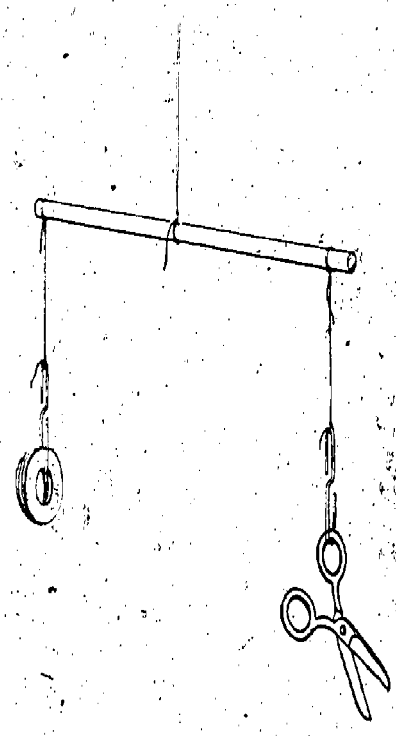
The end of the class came too quickly; the day was over, and we had to stop. The investigation during these classes had not been anticipated in advance, but it followed naturally from the previous classes and proved exciting and rewarding.

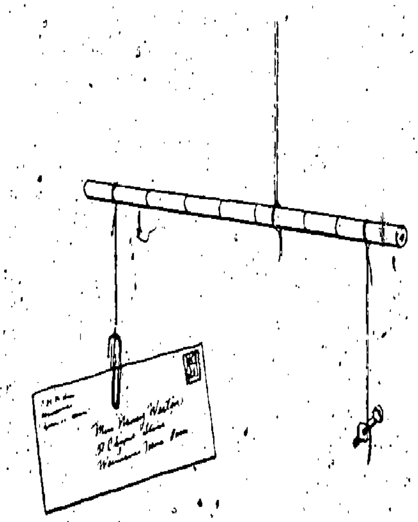
Appendix B

Making Balancing Equipment

A pan balance can easily be made from the balance-beam assembly. It is handy for weighing or balancing many items that can't be hung by clips. The pans are hung by strings and connected to the beam by paper clips. Paper or aluminum pie pans, plastic freezer containers, or empty half-pint milk cartons can be used for pans. You can hang them by threading the string through several points around the top edge of the pan, so that it can swing freely from the clip. An object to be weighed can be put in one pan and washers in the other. (Many activities with pan balances are discussed in THE BALANCE BOOK* from the Elementary Science Study.)

A dowel hung from a string makes a fairly sensitive balance. One way of using this is to balance the dowel and then hang things from the two ends by means of strings and clips.





Another way to use the dowel balance is to hang a one-ounce weight at one end and make pencil marks along the dowel to show the position of the string when the weight on the other end is equal to 1/2 ounce, 1/4 ounce, and other fractional weights; this arrangement can be used for weighing mail or other fraction-of-an-ounce items.

A soda-straw balance is extremely sensitive: Flatten one end of a soda straw, and insert a machine screw in the other. Slide your finger under the straw until you find the balance point, and push a needle through the straw at this point. Suspend the straw on the needle between two glasses, and adjust the balance by turning the screw. On this balance you can weigh tiny objects, such as squares of graph paper, seeds, cereal, or glass beads.

Building mobiles is another activity involving balancing. (For information on this topic see the Elementary Science Study book MOBILES.*)

*THE BALANCE BOOK is available from the Webster Divison of McGraw-Hill Book Company.

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